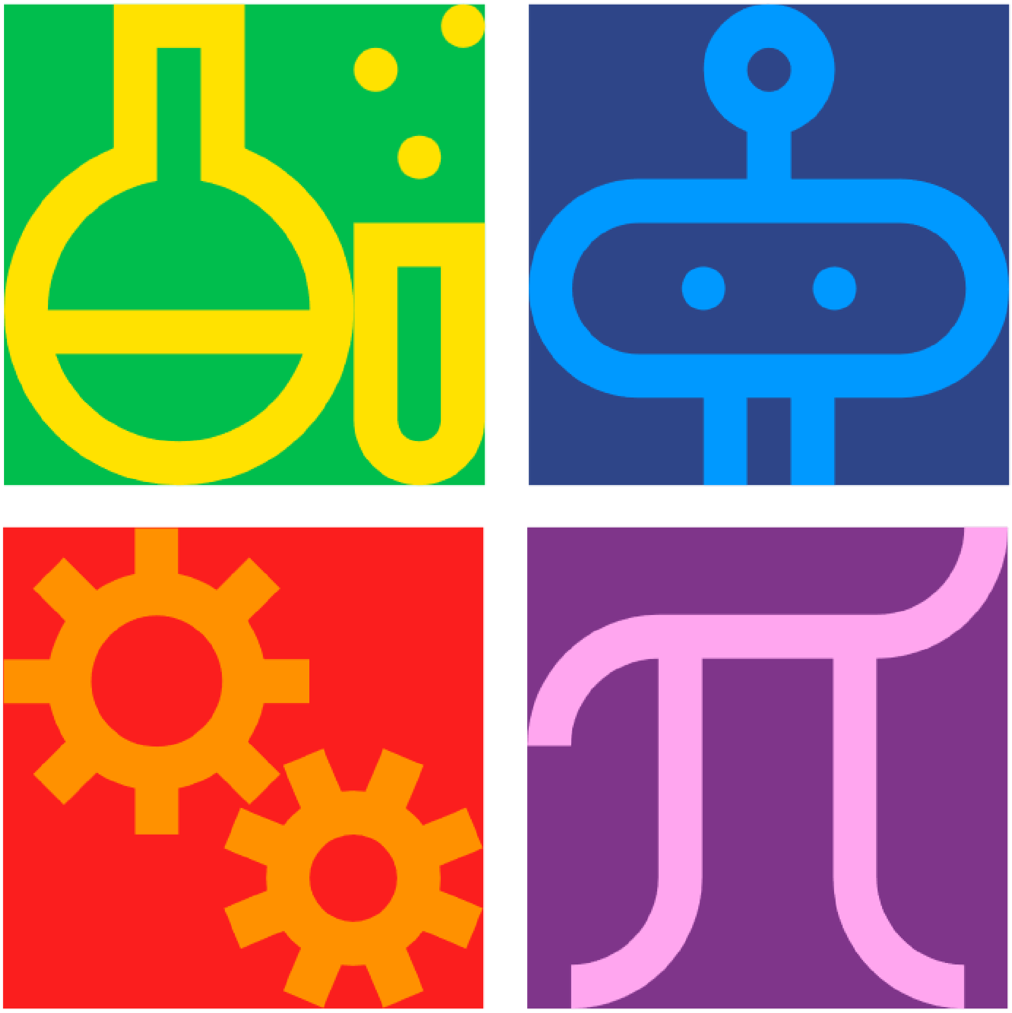
**Matthias Ludwig, Simon Barlovits,
Amélia Caldeira and Ana Moura
(Editors)**

Research On STEM Education in the Digital Age

Proceedings of the ROSEDA Conference



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RESEARCH ON STEM EDUCATION IN THE DIGITAL AGE: EDITORIAL

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In spring 2020, the Covid-19 pandemic caused strict lockdowns of public life all around the world. Also teaching and learning changed massively. Instead of learning in a common place, namely the classroom, the pandemic situation led to a spatial and temporal separation of students and teachers. Hardly surprisingly, this entirely new distance learning situation was perceived as a major challenge by both students and teachers (Hodges et al., 2020).

The need to reorganize the realization of education "shocked teachers at all levels and at the same time inspired them to find solutions to problems they have not encountered before" (Flores & Swennen, 2020, p. 456). Mostly with the help of digital media and the internet (Crompton et al., 2021), educational institutions have developed new teaching methods and found creative solutions for distance learning (Flores & Swennen, 2020).

THE ASYMPTOTE PROJECT

Among other approaches, the ASYMPTOTE project was developed in the context of these distance learning issues (cf. Barlovits et al., 2022). It aimed at creating an adaptive, synchronous and mobile system for online mathematics lessons.

In order to do so, the ASYMPTOTE project (**A**daptive **Sy**nchronous **M**athematics Learning **P**a**t**hs for **O**nline **T**eaching in **E**urope) was carried from March 2021 to February 2023. Within the framework of an Erasmus+ Strategic Partnership, seven institutions from five countries contributed to the project, namely:

- Goethe University Frankfurt, Germany
- Autentek GmbH, Germany
- School of Engineering, Polytechnic of Porto, Portugal
- University of Bielefeld, Germany
- University of Catania, Italy
- University of the Aegean, Greece
- Spanish Federation of Mathematics Teacher Societies, Spain

From a technical point of view, ASYMPTOTE offers two components. In a *web portal* (www.asymptote-project.eu/), teachers can select task examples, so-called learning graphs. It also offers an authoring tool that teachers can use to create learning content themselves. Students work on this content through a *mobile app* which is available for iOS and Android devices. Their progress can be monitored by teachers in the web portal's Digital Classroom feature. The ASYMPTOTE system can be used free of costs and advertisement.

Ludwig, M., Barlovits, S., Caldeira, A., & Moura, A. (2023). Research On STEM Education in the Digital Age: Editorial. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 1–5). WTM. https://doi.org/10.37626/GA9783959872522.0.01

In addition to the technical development, the ASYMPTOTE project also aimed at creating best practice examples for digital tasks and learning graphs. As part of the project, we developed a comprehensive *open database* of learning content for secondary schools and universities. To enable teachers to create their own content to meet the needs of their students, ASYMPTOTE provides detailed video tutorials and an extensive step-by-step user guide.

Pre-service and in-service teachers were trained in the use of ASYMPTOTE in various events: A university course for pre-service teachers was developed and delivered at the Universities of Bielefeld, Catania, Frankfurt and Rhodes (*long-term curriculum*). Furthermore, a *massive open online course* (MOOC) on ASYMPTOTE was conducted in 2022. Other dissemination activities included an intensive study program for university students (Frankfurt, Germany) and an international teacher training (Granada, Spain).

Lastly, the ASYMPTOTE project was continuously monitored from a scientific perspective. Besides an ongoing evaluation of all technical developments, a pedagogical model for ASMYPTOTE was set up in view of relevant literature on teaching and learning mathematics online (cf. Fesakis et al., 2022). Moreover, a *scientific evaluation* of users' perception of ASYMPTOTE was embedded in all project-related dissemination events. First results were presented at the ROSEDA conference and can be found in these proceedings.

THE ROSEDA CONFERENCE

How can STEM education be enriched by means of digital technology? This key question was addressed by the "**R**esearch **O**n **S**TEM **E**ducation in the **D**igital **A**ge" conference (ROSEDA)**.** It took place in Porto from February 23 to 25, 2023, and focused on teaching and learning of science, technology, engineering, and mathematics with digital technology.

Given the great importance of digital technologies in STEM education (cf. Tytler, 2020) and in dealing with the pandemic situation (cf. Crompton et al., 2021) the ROSEDA conference dealt with the following three topics:

- ideas and experiences for the design, conduct, and assessment of online courses;
- innovative approaches of using digital technology in education;
- research on teaching and learning during the Covid-19 pandemic.

During the conference, various approaches to digitized STEM education were presented, shared, and discussed. For all conference contributions, a short outlook is given below. Thereby, we start for all three topics with a short introduction of the invited contributions. All other contributions are grouped in view of their topics.

Ideas and experiences for the design, conduct, and assessment of online courses

In their invited paper, Corlu et al. explore key elements for effective online professional development in STEM education. In doing so, the authors investigate in a case study the implementation of STEM lessons by a teacher after participation in a PD programme.

The conduct and assessment of university is addressed by two papers. Geisen and Zender present ideas for formative assessment of online higher education seminars through a balanced mix of instructor and peer feedback. Läufer and Ludwig show an approach to incorporate 3D printing into university courses for student teachers.

Kleine and Anhalt emphasize the need for building adequate cognitive structures, so-called "Grundvorstellungen", for students' ability to work on mathematical modeling tasks. An approach how mathematics can be learned enactively in distance education is presented by Kleine and van Randenborgh.

Innovative approaches of using digital technology in education

Vaz de Carvalho presents different approaches for technology-enhanced active learning in his invited paper. He shows best practice examples from different STEM disciplines with a special focus on serious games.

Several papers address the development of new tools. Barbosa and Pereira present a mapping tool for teaching and learning descriptive geometry or, more specifically, the Monge method. Stäter et al. present the <colette/> project, which aims to enhance students' computational thinking skills. The Erasmus project LEARN+ is presented by Carvalho and Lázaro. Here, the authors pay special attention to the training of teachers in the use of the MILAGE LEARN+ tool.

The math trail idea is addressed by three papers. Benito et al. discuss the possibilities of combining outdoor mathematics and augmented reality elements for initial teacher education. Cahyono describes how virtual reality can be used to conduct math trails including real-world STEM problems despite of the pandemic situation. Using the MathCityMap system, Jablonski evaluates the role of hints and feedback for outdoor mathematics in a case study.

Two papers present and discuss the ASYMPTOTE project. Caldeira et al. describe how the ASYMPTOTE was used in higher education and how university students experienced the usage of the system. Dos Santos dos Santos et al. compare the automated feedback of ASYMPTOTE and GeoGebra and present an approach to take full advantage of both systems.

Research on teaching and learning during the Covid-19 pandemic

Whether it exists a "positive side" of the pandemic, is discussed by Swidan in view of Covid-19-induced distance. In the invited paper, he presents several technical approaches to help teachers deal with this particular situation.

The papers on teaching and learning during Covid-19 pandemic addressed different educational levels. Caldeira et al. present an approach to dealing with the Covid-19 pandemic in which university students developed short videos on linear algebra. In their paper, Oehler et al. focus on secondary students and discuss ways to enhance computer science projects with distance learning elements.

Two papers evaluate the ASYMPTOTE system in the educational praxis: In a case study with elementary school students, Fesakis et al. evaluate the use of the ASYMPTOTE system in school practice. University courses on ASYMPTOTE in Germany, Greece, and Italy are evaluated by Taranto et al.

Workshops and posters

As a poster, Caldeira et al. present an approach for teaching mathematics with robotics as part of a STEM project. Oehler et al. give an overview of the ASYMPTOTE project and its technical development.

The ASYMPTOTE project is also presented by Barlovits et al. in the form of a workshop. Other workshops by Jablonski et al. and Stäter deal with the MathCityMap and the <colette/> system, respectively.

ROSEDA conference and proceedings

During the ROSEDA conference, researchers from across Europe were able to share experiences from research practices in science, technology, engineering, and mathematics education. They discussed innovative approaches to teaching and learning in the digital age and received numerous suggestions from the STEM community.

We are convinced that, with the conference and these proceedings, we can contribute to the manifold discourse on how STEM education can be enriched with the help of digital technologies. We would like to thank all participants of the ROSEDA conference for the rich and fruitful discussions and wish all readers a lot of pleasure with the multifaceted contributions in this conference proceedings.

Frankfurt and Porto, May 2023

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TOPIC 1

IDEAS AND EXPERIENCES FOR THE DESIGN, CONDUCT, AND ASSESSMENT OF ONLINE COURSES

EFFECTIVE ONLINE PROFESSIONAL DEVELOPMENT: A FACILITATOR'S PERSPECTIVE

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The purpose of this qualitative study is to explore the key elements that contribute to effective online professional development as perceived by facilitators. The research focuses on a specific program aimed at integrating computer science into mathematics education, involving two cohorts of teachers over a period of two years. The lead facilitator, Haynes, has a master's degree in mathematics education and 18 years of teaching experience. The study highlights the importance of sparking interest, enabling participants to teach beyond the curriculum, and offering a personalized and diverse approach to online professional development with a focus on mathematics content. Additionally, the findings underline the value of incorporating STEM education theory through cofacilitation with an experienced teacher education researcher. The findings are further discussed in a keynote speech at the ROSEDA conference.

Keywords: Online professional development, PD facilitators, STEM education.

INTRODUCTION

Professional development (PD) has been extensively studied and conceptualized in educational literature, with varying definitions and perspectives. For example, Guskey (2000) defines PD as a comprehensive process consisting of various initiatives to develop educators' competence, abilities, and perspectives, to elevate student learning outcomes. There is a consensus among scholars that PD encompasses more than just behavioral changes (Borko & Putnam, 1995; Clarke & Hollingsworth, 2002; Guskey & Huberman, 1995) and is a crucial aspect of education that, when approached holistically and from multiple angles, can result in significant improvements in teaching practices and student performance (Borko, 2004; Cochran-Smith & Fries, 2005; Desimone, 2009; Kennedy, 2016; Opfer & Pedder, 2011; Putnam & Borko, 1997; Stewart, 2014). Thus, it is essential to continually examine and comprehend the various approaches to effectively implement PD to enhance educators' professional capabilities, knowledge, and attitudes.

The pedagogy of teacher education is multifarious, with inquiries revolving around the modalities of didactic transformation, the requisites for knowledge, and the means of incorporating acquired knowledge into practical applications (Feiman-Nemser, 2008). The theoretical framework of *how people learn* (NRC, 2000) is widespread in explicating the dynamics of teacher education.

The framework is predicated on four perspectives on learning environments: (a) the learner-centered perspective, (b) the knowledge-centered perspective, (c) the assessment-centered perspective, and (d) the community-centered perspective (Bransford et al., 2007). *The learner-centered perspective* considers that teachers have personal characteristics, cultural backgrounds, experiences, and preconceptions that affect their learning and that effective learning environments for teachers should consider these factors and their relationship with

Corlu, M. S., Kurutas, B. S., & Ozel, S. (2023). Effective Online Professional Development: A Facilitator's Perspective. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 9–23). WTM. https://doi.org/10.37626/GA9783959872522.0.02

teacher learning (Opfer & Pedder, 2011). *The knowledge-centered perspective* focuses on what to learn and why it is learned to advance classroom practices and enact new implementations and includes the components of teacher knowledge such as content knowledge, pedagogical knowledge, and pedagogical content knowledge, the curriculum, tools, materials, and activities planned to be used during the learning process (Shulman, 1986*). The assessment-centered perspective* mainly focuses on feedback as the main focus, through reflective self-assessment and reassessment of learning to give learners new perspectives, guide researchers to understand teacher learning better, and design professional development programs (Desimone, 2009). *The community-centered perspective* focuses on creating collaborative, flexible, and inclusive learning environments where learners can share ideas, questions, and performance evaluations to enhance their learning experience (Kennedy, 2016). To effectively enhance teacher learning, all aspects of the four-component framework must be integrated into professional development design, according to Bransford et al. (2007).

Throughout the corpus of literature on efficacious professional development, scholars have begun to dub specific well-established characteristics as *consensus*. Per this consensus, Wayne et al. (2008) succinctly summarized the definition of effective professional development as follows:

"...intensive, sustained, job-embedded professional development focused on the content of the subject that teachers teach is more likely to improve teacher knowledge, classroom instruction, and student achievement. Furthermore, active learning, coherence, and collective participation have also been suggested to be promising best practices in professional development" (p. 470).

Online professional development

Online professional development (OPD) provides opportunities for teachers to create online communities, comfortable settings, and audio-visual aids (Carey et al., 2008; Elliot, 2017; Trust et al., 2016) and to receive instant help and reflect with other teachers from diverse backgrounds and disciplinary programs (Bates et al., 2016; Carey et al., 2008; Fishman et al., 2013; Trust et al., 2016) which allow them to improve professionally according to their interests and needs (Macià & García, 2016).

Research has found that both face-to-face and OPD have positive effects on teacher change and student outcomes, with little difference between the two forms of PD regarding teacher knowledge and instructional change (Dash et al., 2014; O'Dwyer et al., 2010; Parsons et al., 2019; Fishman et al., 2013; Means et al., 2013; Russell et al., 2009). However, blended professional development, which combines both face-to-face and online components, is more effective than either form alone, as it allows for the benefits of both forms to be utilized (Means et al., 2013).

Coteaching

Coteaching, which involves the collaboration of multiple individuals, has been found to have positive consequences for learners and is advantageous for the professional development of the educators involved (Yabas et al., in press). However, creating an effective coteaching partnership requires a willingness to learn from each other's differences and strengths, establish clear roles and responsibilities, communicate effectively, and negotiate conflicts that may arise (Pratt, 2014; Rytivaara et al., 2019). Additionally, at the in-service teacher education level, a unique challenge of coteaching is the need for partners to shift their self-perception of their traditional roles (i.e., teachers, pre-service teacher educators, or researchers) into that of PD facilitators (Perry & Boylan, 2018).

Research questions

There is a need to update the consensus view on effective PD as suggested by multiple studies (Darling-Hammond et al., 2017; Hill et al., 2013; Wayne et al., 2008). It would be important to consider the perspectives of individuals who work at different levels of PD in order to gain a comprehensive understanding of effective PD. While some studies have collected data from teachers (e.g., Garet et al., 2001; Penuel et al., 2007), there is a lack of research that involves the perceived roles and responsibilities of OPD facilitators (cf. Tekkumru-Kisa & Stein, 2017). Thus, this qualitative inquiry aims to investigate the characteristics of effective OPD from a facilitator's perspective.

METHODS

As a qualitative research method, phenomenology offers a means of gaining deeper insights into phenomena, including aspects that have not been previously explored, by examining how individuals experience and understand the phenomenon and uncovering its essence (Merriam, 2009). This research approach allows for discovering new perspectives and avoiding oversimplifying experiences by only considering obvious theoretical patterns (Spiegelberg, 1965). This study aimed to identify and investigate previously unexplored characteristics of high-quality professional development (PD) and coteaching experiences, along with those already described in the literature.

Context

The context of the present study is the professional development programs provided by the *STEM teacher education and research center* (*STEM center*) at a private university in Turkey. Annually, over 1,000 teachers participate in these programs. The center's most notable program, the *STEM leader teacher OPD program*, offers an 8-month training that certifies teachers as *STEM leader teachers* upon completion. All programs offered by the center align with the *STEM integrated teaching framework* and aim to promote equity, rigor, interdisciplinarity, and relevance in *STEM Education* (Corlu, 2017).

The administrator of the STEM center (*coordinator*) reached out to experienced and innovative teachers (*facilitators*) in the community and invited them to create a short-term OPD program as an advanced follow-up to the STEM leader teacher OPD program. In the second cohort of the programs, the coordinator encouraged all facilitators to incorporate a theoretical component into their program by partnering with teacher education researchers possessing relevant expertise.

The center specified certain requirements for the chosen programs, highlighting the practical application of STEM Education as a best practice and promoting the principles of STEM Education. These requirements included: one hour of synchronous online meetings per week for seven weeks, selection criteria in the form of a letter of interest for external applications (*teachers*) while certified STEM leader teachers would be automatically selected, the maximum number of teachers would not exceed 15, near complete attendance requirement, and a final summative project (e.g., a lesson plan) for participating teachers. Facilitators were responsible for developing a syllabus and collaborating with the coordinator to align the program with the principles of STEM Education.

Teachers had diverse backgrounds participating from all over Turkey, with no restrictions on the subjects they taught, even though they were mostly mathematics teachers in addition to teachers of physics, information technology, and elementary school teachers.

The present inquiry is conducted across two distinct groups or cohorts over two years for a particular OPD program on computer science and mathematics education: During the first cohort, the facilitator was solely tasked with the planning and delivery of the program, entitled *block-based coding applications for mathematics,* while the coordinator actively participated by observing the program's implementation weekly. The first program aimed to capture the relationship between computer science and mathematics, help teachers increase students' motivation to learn mathematics, and design enhanced mathematics learning experiences. The program covered topics such as geometric shapes, art and mathematics, computational art, number theory and coding, and the connection between trigonometry and coding in a Scratch context.

Compared to the first cohort, the design and implementation of the second cohort program were carried out through closer collaboration between the facilitator and coordinator. This arrangement resulted in integrating an experienced teacher and a teacher education researcher as cofacilitators. Consequently, the second cohort was rebranded *from algorithmic thinking to computational thinking*. The program covered a more comprehensive range of subjects, including algorithms, number theory, geometry, computational art, data science, outdoor mathematical and statistical education using Scratch/Snap! and NetLogo for computational thinking. Additionally, the program included perspectives from mathematics education theory such as developing proof and reasoning skills, APOS theory, constructionism, the history of mathematics (e.g., abacists and algorists), computational thinking in math education, and complexity in mathematics education.

The facilitator

Haynes, the 42-year-old leading facilitator of the OPD, holds a master's degree in mathematics education and has 18 years of teaching experience. He currently serves as a mathematics teacher and head of the department at a private school in Istanbul. He has published several books on the application of block-based programming languages. He graduated from a popular English-medium public university in Istanbul with a degree in mathematics. He worked as a software engineer for two years before entering the field of education. His interest in block-based programming languages arose from teaching fifth and sixth graders and led him to self-study and research Scratch. With prior experience designing one-shot PD programs for his colleagues in areas such as 3D design, robotics, and coding applications, Haynes joined the STEM center community after finding the potential impact promising. He was introduced to the theoretical framework and principles of STEM

Education by *Steven*, the coordinator and his cofacilitator in cohort two, whom he did not know prior to joining the community.

Steven is a mathematics education professor. He had extensive experience as a school teacher, teaching subjects such as mathematics, science, and education technologies. In addition, Steve has designed and facilitated several PD programs and has a research interest in STEM teacher education. As the coordinator of the STEM center, he conveyed the principles of the theoretical framework to the PD facilitators and ensured the program's consistency and sustainability through ongoing support. During the second cohort, Steve also served as a cofacilitator with Haynes.

Data collection & analysis

The process of gathering data for both cohorts in the OPD program was a multi-step endeavor aimed at acquiring comprehensive information about the program through the lens of the facilitator. First, a comprehensive review of the literature on effective professional development and coteaching was conducted to achieve this. Second, the syllabi for both cohorts were thoroughly examined, motivation letters from external teachers were inspected, and recorded sessions lasting up to seven hours were analyzed for each cohort. Finally, the lesson plans served as the final summative assessment. Based on these evaluations, semi-structured interview protocols were prepared, each lasting approximately 60 minutes. The interviews were recorded and later transcribed word-for-word to provide a verbatim account.

The interview protocol for the first cohort's OPD program focused on gaining a deeper understanding of Haynes' motivations, journey, and role in the course design and implementation. In contrast, the interview protocol for the second cohort's OPD program explored topics such as decision-making during co-facilitation, and the impact on professional development for teachers and facilitators.

First, the researchers wrote analytic memos and personal reflections on the data (Saldaña, 2013). These notes became the initial codes, which were strengthened by multiple forms of evidence. Second, the codes were grouped into categories and themes, with some codes forming a single category and others forming a broader meaning through recursive or similar codes.

FINDINGS FOR EFFECTIVE PD CHARACTERISTICS

Motivated for mastery: fostering interest and inspiration

Haynes was trying to find passionate teachers eager to learn about innovative teaching methods. So, he delved into the motivation letters of external teachers and later found that those who had already considered the benefits of block-based programming for teaching mathematics were genuinely invested in the subject.

Sadly, Haynes says that many school administrators require teachers to attend PD programs without considering their interest in the subject, resulting in teachers merely obtaining certificates for promotion instead of developing effective teaching skills. He recalls one instance where some applicants briefly wrote about their excitement for PD, but the lack of detail revealed it was to fulfill a school obligation.

I delved into the motivation letters and discovered they were brief. So, I encouraged teachers to add more detail. I aimed to foster interaction. Unfortunately, some teachers approach professional development without giving it much thought. I can share the standard view in Turkey - the administration imposes it. However, as the teacher thinks it over, they become more convinced that it benefits themselves and their students.

However, Haynes saw this as an opportunity to build a connection with the teachers. He encouraged them to add more detail to their letters and fostered interaction. Haynes believes that as teachers reflect on the benefits of computer science for teaching mathematics, they will become more convinced of its value for themselves and their students.

Haynes' ultimate goal is to ignite a spark in teachers and show them the limitless potential of computer science for teaching mathematics in the classroom. He wants to share his experience and inspire others to take their learning into their own hands. With resources like Scratch projects available for beginners, Haynes believes that motivated and curious participants can achieve high-quality teaching of some rigorous mathematics topics in their classroom. According to Haynes, one motivated teacher can lead to a chain reaction and create a community of innovative educators at their school.

Breaking the boundaries of centralized curriculum and fostering creativity in the classroom

Haynes understands the struggles that come with the centralized curriculum and its impact on teachers. The endless cycle of exams, school administrators, parents, and even students policing the curriculum can make the teaching profession feel suffocating. He believes that teaching is made trivial with a set curriculum designed to provide uniformity across the nation's schools and create the illusion of equity for all students. However, Haynes is on a mission to break these boundaries and empower teachers to take control of their teaching.

The curriculum may tell the teacher what to teach, like how to add rational numbers and teach it with strict subheadings, but we have to offer a new path for teachers to explore their passions and creativity in the classroom. It will not be easy, but it is worth the fight to set teachers free and allow them to enjoy teaching in the classroom truly. The curriculum should not limit what is taught...This is also due to the lack of effort [on the teachers' part]. For example, they only explain what they are told to explain. Mathematics is not just about what is included in the curriculum.

Haynes understands that it should be up to the teacher to bring mathematics to life in the classroom. Despite some subjects being touched upon briefly in the curriculum, with regards to computer science, Haynes believes that the limiting type of instructions, such as "don't *do this, don't do that*", set by the curriculum on what not to teach is not productive, particularly for subjects such as mathematics. Instead, he deems a teacher should "*go in the direction of what the class wants*".

Moreover, he recognizes the value of allowing teachers to tap into their creativity and inspiration in the classroom and emphasized the importance of fostering their passion and confidence through the OPD. Haynes elaborates about his thinking that no curriculum can keep up with the rapid technological advancements and how mathematics plays a role in these advancements.

...math is maybe the same as 20 years ago; at least the topics are the same, but computers are not the same as five years ago... like finding the LCM and GCD using a line, but why do we need the Euclid algorithm? They [students] personally do not need it in their daily life, and mathematicians also do not need it much, perhaps only in Diophantine equations. However, computers do need it. That is where the connection to computer science comes in. Solving a problem for a computer is different from solving it in daily life, mathematics, or physics. Each discipline, like physics, chemistry, sociology, and computer science, has a different perspective. The goal is to understand the differences and see that [mathematics] exists in bigger things.

Haynes aimed to empower teachers to produce lesson materials that integrate computer science into mathematics teaching by the end of the OPD. In addition, the goal was for participants to become equipped to create lesson plans that fit their unique situations and incorporate computer science into mathematics. He quotes Conrad Wolfram (Wolfram, 2020) to summarize that computers change the way mathematics is done, so the teachers should adapt:

If math in a world without computers is the same as math in a world with computers, then there's a serious problem here. That's what Conrad was saying about math. Like, if I ask Siri to solve an equation for me, right? I say, find the roots of x^3-3x^2+7x+5, it can do that, right? Now compare that to a world where that doesn't exist. I'm not even talking about just finding the roots of an equation. If the way you explain math is the same in a world with computers and a world without, then that in itself is a big problem.

Diversified and localized online professional development

Haynes makes a compelling case for the value of diversity in PD content, recognizing that every participant brings unique perspectives and needs to the table. To ensure that everyone could find a way to connect the content of the OPD to their classrooms, he offered a variety of examples and activities, as well as supplementary resources. On the one hand, he urged participants to embrace a wide range of content, as it increases the chances of capturing everyone's interest and may even lead to new opportunities. On the other hand, Haynes emphasized the unifying power of art and its impact on human experience, showcasing the connection between mathematics and art in the context of the OPD.

An algorithm is beautiful. People often say *how beautiful* when they see it. ...everyone has a sense of beauty. Art is a place where everyone can come together easily. It is interesting, like a telepathic experience. In Turtle art, everyone says it is beautiful when you draw something. Everyone finds something different and understands something else when they look at it.

Haynes stresses the significance of understanding the backgrounds of teachers in order to be an effective PD facilitator during design and implementation. He notes that while many online resources, such as YouTube videos and online courses, are available for teachers in the US or Europe, they may not always apply to Turkish teachers. The issue is not language barriers but how teachers learn and do their profession. Haynes emphasizes that knowing the unique characteristics of teachers is crucial for delivering effective OPD.

I could see teachers' strengths and weaknesses over the years [as head of both mathematics and information technologies departments]. So, we designed the program to fit the needs of Turkish teachers. We considered the local education system, the subjects the teachers teach, their knowledge levels, the knowledge level of their colleagues at schools, the students and curriculum.

Haynes also weights the significance of a secure environment for open communication, as some participants may hesitate due to their concerns. To enhance interactivity, it's vital to establish a relaxed ambiance where individuals feel at ease to express themselves. This is particularly evident in the case of mathematics teachers in Turkey, who are subjected to societal pressure and expected to be error-free.

In contrast, the coding world operates differently. Even experienced programmers at leading companies like Google are less afraid of making mistakes. It's widely accepted in this field that errors are a natural part of writing code. People often begin by copying and pasting code from Google, a common practice. Mistakes are not perceived as a hindrance in this industry. However, this is not the case in mathematics, where mathematicians are confident in their abilities and make fewer mistakes.

Haynes highlights the importance of diversity and customization in OPD content, considering participants' unique backgrounds and needs and the impact of art in bringing everyone together. He also emphasizes the significance of understanding the local education system and teachers' knowledge levels and concerns for effective OPD design and delivery.

Pragmatic approach

Haynes stresses the significance of considering that teachers who participate in professional development are eager to acquire practical knowledge that they can immediately apply in their classrooms. This is why he went above and beyond to incorporate hands-on examples that teachers can utilize as-is or tailor to fit their unique classroom environments.

Teachers think about how the knowledge they gain will impact them immediately, not in the long term. [In a successful OPD program], the most important thing is for teachers to leave with both a full head and a full pocket, ready to put their new skills into practice in the classroom.

Haynes faced challenges in creating the syllabus as he revised it multiple times. The issue was figuring out the most important, relevant, interesting, and helpful information to include. According to Haynes, the critical factors in determining what to include in the syllabus were whether the teacher can use the information directly in the classroom and if the teacher can adapt the information to different contexts. Haynes narrowed down 30 pieces of information to just six that would be covered in a seven-week OPD program. The difficulty was determining which six were the most critical and relevant and presenting them in a coherent context. The primary considerations were whether the teacher could adapt and use the information directly.

Haynes initially favored Scratch over Snap! during the first cohort due to the community surrounding Scratch, which offered numerous ready-to-use resources for teachers. However, after a year, he confidently switched his preference to Snap! for the second cohort.

When I talk about Scratch, I find that it is not the best language for explaining math and other subjects. At the time, I wanted to use Snap, but its version was not stable yet and had some bugs. The website was not even opened, and there was no community to share problems or bugs. You had to write directly to the developer. That is why I did not use it in the first year. Later, when the website was up, and a community was formed, they started fixing the bugs. I would still choose Snap! now, because it is a more advanced tool. The main difference is that Scratch does not have factorials or return functions and is limited for mathematics. However, if there is no community, how can we continue to learn? I learned on my own, but what about others? With just 3-5 people, nothing will happen. But with 60 million people or 1 million people, something can happen. It is great when you do a search and find someone who has already thought about it, and you can say *Aha, someone has already thought of this*.

Haynes stresses the importance of considering information's immediate application and adaptability in creating effective OPD programs. He also highlights the importance of carefully considering design decisions, such as incorporating a supportive community, to enable teachers to use the acquired knowledge in the classroom effectively.

Discovering the power of mathematics

Haynes is determined to ignite a passion for mathematics during the OPD. He firmly believes that grasping the subject requires a holistic approach. Through his computer engineering studies, Haynes discovered the true power of mathematics, seeing its practical applications come to life. He understood that not all learning might seem useful now, but if teachers in the OPD could uncover the connections and big picture, it could break down preconceived notions and deepen teachers' understanding of mathematics.

For example, I realized...about mathematics while taking computer engineering classes and thought, "Why didn't they teach me this before?" I need to see a real-life application of polynomials; why are they useful? I have seen rings and objects in space, but what is the practical use of polynomials in real life? Can you give me one example? In computer science, I learned about the five keys for nuclear missiles, which are related to encryption and decryption. Here, I learned that polynomials were used for that. This was discovered in the 1960s, but polynomials are actually very old. I am not saying that polynomials are only used in this context but seeing even one example changed my perspective on mathematics. Suddenly, I understood why I saw those rings [in abstract algebra]. Of course, not everything we learn will be directly useful to us in our jobs. We cannot be that pragmatic. Not every line we read will be useful. But when I see the big picture and understand the connections, it breaks down my prejudices. Unfortunately, many of our teachers do not learn mathematics through practical applications.

Haynes believes that having a shared focus, in this case, mathematics, for participants to discuss is key to their engagement during the OPD. Participants' confidence level in the mathematical concepts affected their ability to discuss and interact. The better the comprehension of mathematical concepts by teachers, the higher the level of interactivity during the PD. Haynes highlights that deeper understanding leads to increased interest.

Haynes stresses the significance of considering participants' prior knowledge while designing the OPD. He points out that previous mathematics content knowledge can directly impact participants' cognitive load.

Teachers had mixed feelings about the mathematical examples in the OPD. They liked the examples but said they had trouble understanding them on their first try and had to watch the [recorded] videos multiple times. It can be hard to learn new concepts, especially if you are at an elementary level for both math and coding. Also, doing two things at the same time can be a challenge.

Haynes notes that working with coding applications can be challenging for both teachers and students, who are often new to the field. To overcome this, the facilitator, Haynes, aimed to balance basic and interesting examples to keep motivation levels high. He started with simple activities and gradually incorporated more challenging yet engaging examples throughout the program. Despite all, he is particularly aware and concerned with the level of content knowledge of middle school mathematics teachers and information technology teachers.

For example, information technology teachers do not necessarily graduate with much mathematics knowledge. Forget about mathematics; there is no algorithm course in their teacher education. There is a coding course but no algorithm course. Yet, it is one of the most challenging classes, even for computer engineers...because it is not just about doing things in sequence. Our concern is how to make the computer do it.

Haynes believes that understanding mathematics is crucial for participants in OPD and that their prior knowledge of the subject can impact their learning experience. He believes in taking a holistic approach to learning mathematics and that seeing its practical applications can change participants' perspectives and break down their prejudices. He also stresses the importance of having a shared topic and the level of confidence in mathematical concepts for participant engagement and interaction.

Balancing time

The duration of OPD is an important factor for Haynes in achieving goals. Depending on the PD's aim, the program's length can vary. For example, if the goal is to provide deep expertise in a subject, a longer duration is required, while a shorter duration may suffice if the aim is to raise awareness. The program length of the OPD was imposed on the facilitator by the coordinator, and the program's design needed to reflect this.

Haynes suggests that attention span should be considered in OPD duration and suggests an active-passive balance to prevent loss of attention. He knew that the suggested duration is 80 minutes, beyond which the cognitive load may become too demanding. Considering these factors, OPD programs can be designed for maximum impact and effectiveness.

Haynes stresses the importance of time in learning and suggests that multiple PD sessions are necessary to understand and apply the material thoroughly. He believes that participants should have at least three to four hours per week to repeat and study the material and highlights the impact of PD workload on participants, who often spend extra time re-watching sessions and internalizing the material.

In addition, Haynes acknowledges the importance of creating a comfortable and interactive environment for participants. The first few weeks may be hesitant as participants adjust to the new environment, but as they become more comfortable and interactive, they will be more participative.

Haynes emphasizes the need to consider the work schedule of in-service teachers who are often the OPD participants. The duration of the OPD should not disrupt their work routines. Haynes considers this by putting himself in their shoes and empathizing with their situation.

Online & offline engagement: finding the best of both worlds

Haynes values in-person interaction over recorded videos (e.g. YouTube or other MOOCs) for better human connection and engagement. He likes the interactive nature of synchronous lessons and believes they enhance discussions and self-reflection. Despite this, he acknowledges the importance of having a structured platform for sharing and managing materials and feedback. He faced challenges with managing final projects and sought a web platform to solve them.

Haynes plans to combine recorded videos with in-person sessions and assign tasks based on the videos for discussion in the coming years. When participants requested homework for motivation and self-reflection, Hayes agreed, saying even adults need homework to revise. Haynes emphasizes the importance of allowing immediate application and reflection for stronger comprehension.

Harness the power of theory through cofacilitating

Haynes stresses the importance of having a well-defined goal and how using *understanding by design* (Wiggins & McTighe, 2005) framework helped him maintain that focus during the OPD design. He notes that for all educators, it's crucial to have a clear objective, like learning about computer science or incorporating mathematics, to determine their end goal.

> I have used the backward design [when designing the OPD]. First, what are my expectations at the end? Where am I headed? Because now, fields like computer science, which is a broad field, especially now with math entering, they are infinite worlds. If you start with this, there are many places you can start from. And there are even more places you can finish. So, I will start with 100 doors, but there are millions of exits at the end. Where will I come out? Otherwise, we will be lost; it is an easy area to get lost in. ... we are talking about two separate, huge disciplines. They are like DNA spirals, yes, exactly like that. They are not the same, but they overlap a lot. They support each other. So, where we will come out is our most challenging part.

During his preparations for the OPD, Haynes reached out to several experts in the field, including Samuel from a university in the USA who had developed the mathematics portion of a programming course. Haynes and Samuel formed a connection as they discussed their experiences and knowledge in the field. Samuel shared his own work and insights with Haynes, who greatly benefited from their conversations and was able to gain a better understanding of how to approach incorporating computer science into mathematics from theoretical and practical perspectives. Haynes' research and consultations, including his interactions with Samuel, helped him in developing the OPD for the first cohort of teachers, giving Haynes a strong foundation to build upon.

For his collaboration with Steven during both cohorts, Haynes was pleased that Steven treated him as a peer, exchanging ideas and viewpoints despite Steven's academic background. This mutual respect only enhanced their discussions.

> Steven was breaking the mold of what I thought academics were like. He challenges my perception and the stereotype that academics are disconnected from the reality of schools and teachers. Normally, if I were working with a typical professor, I don't think it would be as fulfilling, but with him, the experience has been much more enlightening.

The bond between Haynes and Steven was further fortified in the second cohort, as they collaborated in an exhaustive examination of objectives and content. Haynes acknowledges the essentiality of Steven's role as the architect of the OPD and views himself as the principal craftsman. Haynes believes that the architecture of the OPD is of paramount importance and is a determinant factor in the program's success.

Throughout the implementation phase, Haynes was cognizant of the influence of Steven's past experiences and the utility of incorporating the findings of other researchers, as well as incorporating mathematical education theories to enhance the practical applications of the program. Haynes perceives that the joint effort between them significantly enhanced the efficacy of the OPD.

This OPD is something that needs to be constantly refreshed. I have been saying *this is how it was here, this is how it was there*, but why? Steven has more [theoretical] experience in this because he has a broader perspective. For example, when preparing for the second cohort, we looked at last year's lesson together. I was a bit more focused on my immediate environment, but he had a broader perspective. I may have provided the main material for the topic, but in the end, it was the theoretical perspective ...that made the work beautiful. I felt [what I was doing] was right and good but now I knew why it was good. Because without it, it was just a rough building, but now it's a well-finished and beautiful building.

Haynes believes that computer science, compared to mathematics, is still relatively young in terms of its history. As a result, there are still some aspects of the integration of computer science in mathematics education that have yet to be researched. Theoretical discussion within the OPD has given Haynes a clear idea of where the field is headed in the future, and Haynes sees this as the most important aspect, as it helps guide their next steps as teachers and educators.

Haynes states that student learning through teacher effectiveness in the classroom are the primary objectives of the OPD. Haynes is aware of several PDs where computer engineers explain technical concepts, but believes they are not effective. Haynes emphasizes that success of this OPD lies in finding ways to integrate technical concepts, such as coding, into mathematical teaching strategies. Haynes feels that this approach blends theory and practical applications in both mathematics and computer science education, creating a unique and effective path for future OPDs. He provides some insights from OPD sessions:

As a teacher, I find it effective when I can bring all the pieces together. That is true but I need an example [how theory and practice are integrated]. Doing things separately and trying to fit them together later is not only difficult but can also be confusing. The key is to add the right ingredients at the right time to make it complete. Steven could observe student reactions in real-time during the OPD and make necessary comments from theory. This kind of integration is what makes our OPD valuable. At the end of each class, Steven could summarize the main points to bring the broad picture in front.

According to Haynes, the integration of computer science into mathematics education still has a long way to go. He believes that theoretical discussions in the OPD program play a crucial role in shaping its future direction. He recognizes the iterative nature of OPD design, where feedback and evaluations from all stakeholders, including academicians, facilitators, teachers, and if possible, students, should inform each step in the program's development. Haynes attributes his understanding of the importance of having a broader perspective to Steven and the theories presented during the second cohort.

CONCLUSION

Results show that the elements of consensus view for effective professional development—content (*discovering the power of mathematics*), duration (*balancing time*), collective participation (*online & offline engagement: finding the best of both worlds*), active learning (*breaking the boundaries of centralized curriculum and fostering creativity in the classroom*), and coherence (*diversified and localized online professional development*)—are consistent with the findings from this study on OPD from a facilitator's perspective. In addition, this study reveals some new insights regarding effective PD characteristics in *motivated for mastery: fostering interest and inspiration*, *pragmatic approach*, *harness the power of theory through cofacilitating*.

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FORMATIVE ASSESSMENT IN ONLINE COURSE – IDEAS AND EXPERIENCES

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Abstract. *Formative assessment is known for its positive effects on learning outcomes. However, there is still a need for research on the use and successful implementation of this approach. In this article, a university online course about math trails for pre-service teachers is presented, who took the key strategies of formative assessment into account. The concept and the implementation of the course will be presented especially with regard to these key strategies showing how formative assessment can be used and what potential lies in it.*

Key words: Formative assessment, math trail, online course.

THEORETIVAL BACKGROUND OF FORMATIVE ASSESSMENT

When planning a lesson or a course - e.g. for students at school or at university - the didactical concept of constructive alignment has to be considered to coordinate teaching and learning methods, forms of examination and desired learning goals. Therefore, three aspects must be considered (cf. Biggs & Tang, 1999; see also Figure 1):

- Learning objectives: What should the student know or be able to do?
- Learning activities: How does the student reach the final level?
- Assessment: How are knowledge and abilities measured?

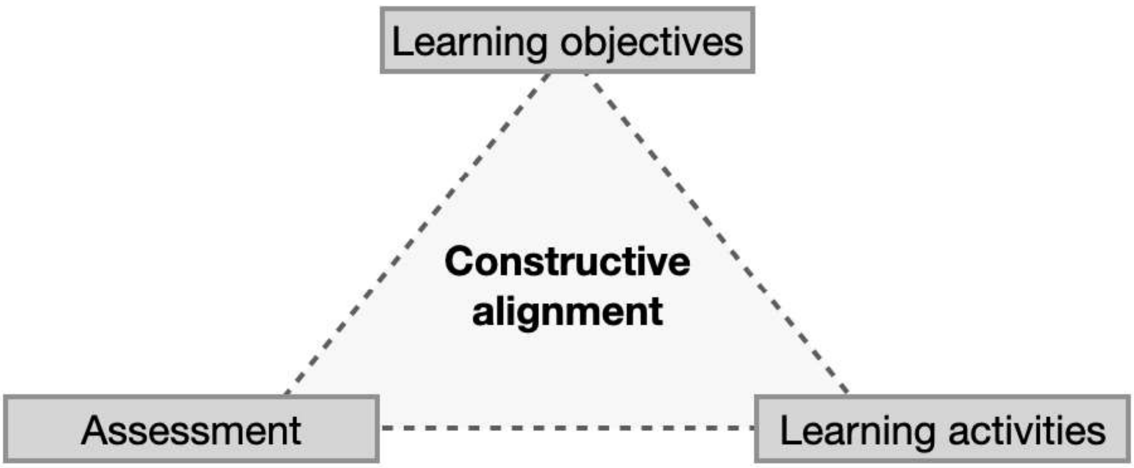

Figure 1: Constructive Alignment based on Biggs & Tang (1999).

With regard to the aspect of assessment, Gerick, Sommer & Zimmermann (2018) distinguish based on Schaper and colleagues (2012) and Knight (2001) result-oriented and process-oriented or summative and formative assessment.

Geisen, M. & Zender, J. (2023). Formative Assessment in Online Course – Ideas and Experiences. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 25–32). WTM. https://doi.org/10.37626/GA9783959872522.0.03

Summative assessment is mostly about grading at the end of a learning unit at school or at the end of a course at university, for example, through a written or oral exam (cf. Sadler, 1989; Brookhart, 2010; Cizek, 2010; Maier, 2010; Gikandi, Morrow & Davis, 2011). Formative assessment, on the other hand, takes place parallel to the learning unit or course to support the learning process and to improve individual learning (cf. Brookhart, 2010; Cizek, 2010; Maier, 2010; Gikandi, Morrow & Davis, 2011). Further, formative assessment enables teachers to adapt learning opportunities to the needs of the respective learners (cf. Black & Wiliam, 2008). Wiliam and Thompson (2008) describe five key strategies of formative assessment in terms of school learning (see Figure 2):

	Where the learner is going	Where the learner is right now	How to get there
Teacher	Clarifying learning intentions and criteria for success	Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding	Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	Activating students as the owners of their own learning	

Figure 2: Aspects of formative assessment based on Black & Wiliam (2008).

Concerning formative assessment at school, Schütze, Souvignier and Hasselhorn (2018) identify a need for research on the use and successful implementation of this approach, but this aspect should also be explored regarding university education. For this reason, this article focuses on formative assessment to provide new ideas and experiences in terms of an online course in mathematics for pre-service teachers at university regarding the key strategies described above.

DISCOVER MATHEMATICS OUTSIDE WITH MATHCITYMAP

Mathematics can be discovered everywhere, and extracurricular learning locations can be a useful addition to regular mathematics lessons at school and enrich them (cf. Scherer & Rasfeld, 2010), for example, concerning primary experiences or active construction processes (cf. Sauerborn & Brühne, 2009). As part of math lessons, mathematical trails can be implemented (cf. Shoaf, Pollak & Schneider, 2004), which means that mathematics can be discovered outside at different locations by working on tasks where it is necessary to collect data at the respective locations (cf. Ludwig & Jablonski, 2020, S. 29). For example, the area of objects outside can be calculated, such as the back of a skate ramp (see Figure 3).

For a long time mathematics trails have been used and only the motivational aspects were researched (Gurjanow et al., 2020), but they held the potential to also positively influence the learning outcome (Cahyono, 2018, Zender, 2019). To get the most out of a math trail, the MathCityMap project was founded. In the context of the MathCityMap project such mathematics trails were connected to new technologies such as smartphones and the Internet (cf. Jesberg & Ludwig, 2012), and the MathCityMap app (short MCM app) was developed including a web portal and an app for creating and implementing math trails (cf. Ludwig & Jablonski, 2020). In this way, not only the above-described advantages of discovering mathematics outside are used, but more applications and references to reality can also be taken into account.

Figure 3: Picture by Anna Albrecht.

FORMATIVE ASSESSMENT AS PART OF A MATH TRAIL COURSE FOR PRE-SERVICE TEACHERS

Classification and design of the course

In the summer of 2021, two online courses about mathematics trails were offered for pre-service teacher students at the University of Koblenz-Landau (cf. Geisen & Zender, 2022). The students taking part into this course all lived in the area of Koblenz, which made it possible for them to meet outside in small groups. This is based on a course developed by Ludwig, Gurjanow and Barlovits at the University of Frankfurt (part of the EU-Erasmus+ project MoMaTrE), which was adapted by adding new elements. Besides, the course was held entirely digitally. Forty-five each took part in both courses, so ninety students of elementary school mathematics in total, most of which were female.

The main goal of the course was to enable the students to create mathematics trails. Therefore the course was subdivided into the following learning units:

- The students developed the theoretical background for extracurricular learning in mathematics in small groups (e.g. Scherer & Rasfeld, 2010; Sitter, 2019).
- In this small groups, the students developed the theoretical consideration of relevant general didactic topics in connection to math trails (e.g. digital media, group work). And every student took a photo of an extracurricular learning place for mathematics that can be connected to any other subject.

- The students developed relevant mathematic didactic topics in small groups (e.g. task design, modelling, measurement).
- Every student did a mathematical walk through her/his neighbourhood and photographed objects where mathematics can be discovered.
- The lecturer introduced the project MathCityMap and the MCM app.
- The students walked a mathematics trail with the MCM app selected by the lecturer in small groups and worked on tasks outside so that they could gain experience. Afterwards, they worked on two reflection tasks in this regard: On the one hand, they reflected on what they liked and did not like. On the other hand, they chose the two most difficult tasks of the mathematics trail and analysed their own hurdles and possible difficulties of learners.
- The students used the MCM app and developed mathematical tasks for three objects.

All tasks and pictures had to be handed in by using a Learning Management System. At the end of the course, asynchronous oral exams were held (cf. Geisen & Zender, 2022).

Concerning the methodical design, the students had to form small groups and meet in person (outside) at the beginning of the course in order to work on all the above-mentioned assignments together (see below). In addition, working in small groups should demonstrate the positive aspects of cooperative collaboration and have a lasting impact on the students. Especially with regard to the relevance of the cooperation of teachers in terms of their professionalisation and further development of competencies and convictions (e.g. Arndt & Werning, 2013) as well as concerning a relief function (e.g. Trapp & Ehlscheid, 2018). Every group work was reviewed by other groups of students and by the lecturer. In the next section, examples of such reviews were described as well as drawing the connection to the key strategies of formative assessment.

Implementation of formative assessment

At the beginning of a course, it should be self-evident to explain, share and understand the learning goals. Therefore, all goals, especially the main goal to enable the students to create mathematics trails, were of course communicated throughout the seminar. However, the focus of this article is on the other key strategies (recording the (individual) learning status through discussions, tasks or questions, giving helpful feedback, activating learners as being responsible for their learning process and activating learners as resources of learning for each other) which are shown in the following illustration and are interwoven:

The students worked out relevant mathematic didactic topics independently in small groups to create a five-minute podcast (a short audio play). For this, they each chose a topic from various given topics, whereupon two other groups reviewed each podcast. In addition, each group formulated three questions about their topic, the answers to which were used as a work assignment for the other groups.

After the theoretical input, the students were asked to take a mathematical walk through their neighbourhood and photograph objects where they could discover mathematics. The students made their photographs available within the framework of the learning management system OLAT, and the other students were supposed to reflect on the connection between mathematics, the depicted object and a real-world reference on the one hand and possible tasks about the posted photographs on the other hand. One student photographed various objects on a playground, which, among other things, refer to the geometric shape of the triangle (see Figure 4). In this respect, discussion occasions are, for example, the use and function of different types of triangles (e.g. acute and obtuse triangles) as well as the change of the angles of the triangle in relation to the slide experience.

Figure 4: Picture series "Doing mathematics outside, ideas for the playground" by Lea Nisius.

As part of the last practical work assignment, the students registered for the MCM app and developed mathematical proposals for three different objects outside, which the seminar lecturer reviewed online via the MCM Webortal and the review function. After a revision phase, a new trail was generated with these nine objects (three objects each from the three students in the small group), again reviewed by another small group by trying the trail at the site and then again online by the lecturer.

Quotes from the evaluation at the end of the course:

- "I found the exchange about the results and the access to the results of the other students very helpful, because I had such a repertoire of possibilities and solutions. In addition, it is helpful that materials are always accessible."
- "All units of the seminar built on each other. I also found the proportion of theoretical knowledge and practical application very balanced. We received prompt individual feedback on the tasks with additional suggestions. Despite the purely digital teaching, we were able to exchange ideas with each other and gain practical experience. The chosen goal of a completed trail was very motivating to deal with the content in depth, as it had a purpose."
- "The transparency and the common thread that ran through the event are to be commended. I was pleased with the feedback on the tasks handed in."
- "I was very pleased to receive feedback on my submissions. Unfortunately, that is only the exception and not the rule."
- "The structure of the seminar was very good, as you were introduced step by step to the creation of your own math trail."

Judged by the voices of the students, the formative assessment worked out well. As they state, the learning goals were transparent, clear, and therefore motivating. Throughout the intense work in groups of three, the students had discussions and exchange and they recognize the others as resources, they have been activated to give and receive peer reviews of their tasks. In addition, there was the feedback from the expert. All feedback were helpful, and the students even liked it to get feedback. They had joy in learning.

CONCLUSION

The course specifically aimed at the questions where the learner is right now and how the learner reaches the goal. These questions are identified as two aspects of formative assessment (cf. Black & Wiliam, 2008). Giving feedback was the most essential part of the seminar. Feedback came from peers or/and the lecturer as an expert in the field. Since every task had to be handed in, the lecturer was aware of the difficulties and problems of the students, and so the students could be addressed to a special degree. The knowledge about this also influenced the process of the learning-accompanying feedback. The format of an online course opened up the possibilities and showed the needs of these intensive review processes, especially by the lecturer. As the students formed groups, they were responsible for the learning process of their group. Thus, they were also responsible for their own. Peer Assessment is an integral part of the seminar. In many cases, the peers correct the others in the first step, and in the second step, the lecturer gives feedback. Also, working in groups of three the whole time does help to recognize the others as resources for learning. Furthermore it should be mentioned, that the MCM web portal provides good opportunities for feedback, such as creating groups to share, work and comment on common tasks for peers and the review process to comment on handed-in tasks by the expert.

From the point of view of the lecturer, formative assessment was in the presented course a good opportunity to accompany and support the students. It can be assumed that the large amount of feedback during the semester also helped the students with the exam at the end. Overall, it can be promising, where the implementation is of course demanding and complex (cf. Schütze, Souvignier & Hasselhorn, 2018).

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MODELING TASKS UNDER THE PERSPECTIVE OF 'GRUNDVORSTELLUNGEN'

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Abstract. In automated training in mathematics, as in the ASYMPTOTE project (Adaptive Synchronous Mathematics Learning Paths for Online Teaching in Europe), modelling tasks are a challenge in their creation and technical implementation. In modelling tasks, working with mathematics is concretised in the application area. Mathematical work is understood as a process of modelling: First, mathematical models are derived from a real problem; then the mathematical model is solved; finally, the mathematical solution is interpreted with regard to reality and the original problem is validated by the solution. This process focuses on the transition between the reality and the mathematical level. This paper focuses on this transition and its requirements and explains design principles of modelling tasks using examples from proportion and percentage calculation.

Key words: Grundvorstellungen, mental models, process of modeling.

MODELING AND APPLICATION

Starting with the assumption that competences can be learned, the question of different types of learning environments and the understanding of education at school is more interesting than mathematical precision. Klafki (1991) describes education as the ability of self-determination, where all-round education consists of three determining factors: (a) an education for all, (b) a general education and (c) an education with a focus to more general aspects. This aim of an all-round education, which was established by Winter (1976a, 1976b, 1996) in the area of mathematics conveys mathematics in three basic experiences. These experiences can be characterized as (E1) application orientation, (E2) structure orientation and (E3) problem orientation (cf. Blum & Henn, 2003; Winter, 1996). Thereby, 'application orientation' does not directly mean the preparation for specific situations in life, but rather, the possibility of a basic insight into nature, society and culture. 'Structure orientation' focuses more on the analysis of mathematical objects in relation to a deductive view of the world. The problem orientation on the other hand, emphasizes the acquisition of heuristic abilities to recognize and use samples in problem solving processes. However, these three aspects are connected with each other. Mathematics at school should provide the prerequisites to a basic mathematical knowledge in order to gain insights into various contexts of life in a reflective und understandable way. This view supports the ideas of Freudenthal (1973, 1981, 1983), whereby the arrangement of coherences is an essential aim of mathematical teaching at school. From these ideas, an understanding for mathematical competence as an individual characteristic has been developed according to mathematical literacy, which studies such as PISA are based on (OECD, 2022). This paper will focus on the requirements and design of modelling tasks, as these represent a separate task category in technical training exercises such as in the ASYMPTOTE project (Adaptive Synchronous Mathematics Learning Paths for Online Teaching in Europe), which must be implemented individually from a technical point of view (Oehler et al., 2023).

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The mathematical process of modeling

For modeling tasks, the view of mathematical literacy stands out due to an explicit application orientation. Grasping mathematical concepts should be taught by connecting it to real problems and how to use this acquired knowledge in real situations later in life (cf. Griesel, 1976). To solve a real-life mathematical problem, mathematical work can be understood as a process of modeling (cf. Blum, 1996). During this process, there can be separate different phases: (1) At first, the complexity of the real situation has to be focused to the specific problem in hand. You then get a model of reality. (2) This real model has to be transposed to a mathematical model on a mathematical level. (3) The mathematical model is solved and you get a mathematical result. (4) Finally the mathematical result is translated with a view to reality. Figure 1 shows those phases of a modeling process.

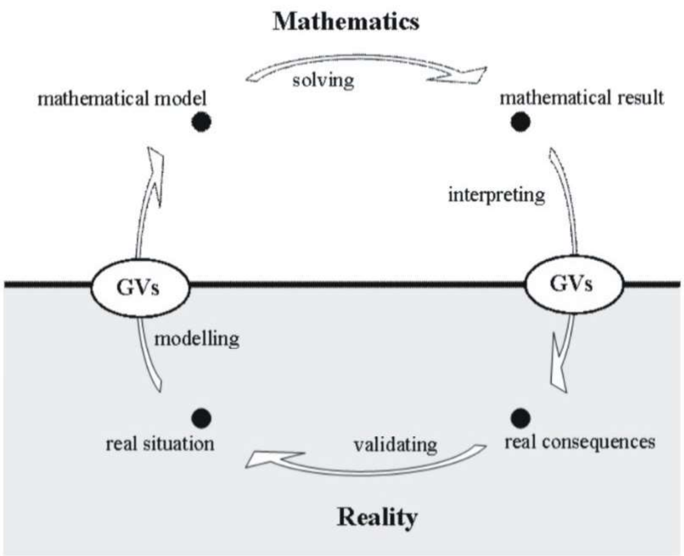

Figure 1: The process of modeling with the integrated concept of 'Grundvorstellungen' (GVs) (vom Hofe, Kleine, Blum & Pekrun 2005, p. 3)

The circulation indicates that the process should be learned by repetition due to the fact that mathematical models have to be modified or compared with other models. The correspondence of the perceived world with the mathematical model indicates the understanding of an individual in regard to nature, society and culture (cf. Winter & Haas, 1997). For further exposition of mathematical competences it is important to look more closely at the transition between reality and mathematics which play a decisive role in the description of mathematical literacy in the following section.

In this process of modeling, the transition between reality and mathematics represents a central task; a real situation is modeled, to the mathematical level and, the mathematical result is interpreted with respect to the real consequences. At first, we need mental objects of mathematical concepts for these transitions.

"I have avoided the term concept attainment intentionally. Instead I speak of the constitution of mental objects, which in my view precedes concept attainment and which can be highly effective even if it is not followed by concept attainment." (Freudenthal, 1983, p. 33)

The construction of such cognitive structures is described as the formation of 'Grundvorstellungen' (short "GV", e.g. vom Hofe, 1995, Greefrath et al., 2016, Salle & Clüver, 2021). This formation is indicated (a) by recording the meaning of new concepts about known structures, (b) by the construction of mental objects which represent the concept, and (c) by the application of new situations. The formation contains both the expansion and change of existing 'Grundvorstellungen' and the construction of new 'Grundvorstellungen' (cf. vom Hofe, 1995). 'Grundvorstellungen' therefore, describe fundamental mathematical concepts or methods and its interpretation into real situations. They describe the relations between mathematical structures, individual psychological processes and real situations. Kleine (2012) points out three features of this concept:

a) A clear relationship cannot exist between mathematical objects and specific 'Grundvorstellungen' because usually, mathematical objects are represented by several 'Grundvorstellungen' which are connected to each other.

b) One can distinguish 'Grundvorstellungen' in two ways: On the one hand, there exist primary 'Grundvorstellungen' which begin usually before mathematical instruction and these stand out due to concrete actions and concrete operations (e.g. the 'Grundvorstellungen' of the sum at a). On the other hand, secondary 'Grundvorstellungen' are developed during the time of mathematical instruction, which is indicated especially by mathematical representations (e.g. the 'Grundvorstellungen' of a function concept).

c) 'Grundvorstellungen' are neither fixed, nor used universally but are dynamic and are developing within a networked mental system. The necessity for the development results in a varying range of validity: If 'Grundvorstellungen' are sustainable in one mathematical area, they must be extended in another area. For example, 'Grundvorstellung' of multiplication when using different numbers for the second factor has separate results. With natural numbers the product is always higher than the first factor; with fractional numbers however, the product can be higher (2nd factor >1) or lower (2nd factor <1) than the first factor.

These characterizations point out, that 'Grundvorstellungen' can not be directly studied and require the need to be aware of the different types of behavior. This point of view marks the descriptive aspect of the concept: Through the analysis of individual behavior (e.g. at school, interviews, exams) the aim is to reconstruct the existing 'Grundvorstellungen' of mathematical objects. In contrast with this idea there is the normative aspect, whereupon 'Grundvorstellungen' are used as guidelines for the construction of mental objects of mathematical contents. The first aspect questions which 'Grundvorstellung' has been activated by a student; the second aspect questions which 'Grundvorstellung' has to be formatted by the student. If one compares the existing with the desired 'Grundvorstellungen', we have the idea outcome of an agreement. In many cases you can observe a deficit. This is a central topic of didactical research in the field of 'Grundvorstellungen'. Vom Hofe (2003) establishes the connection between basic ideas and basic education.

The previous explanations are intended to show that modelling tasks have a complex profile of requirements that must be taken into account when designing and evaluating tasks. It is therefore all the more important to structure modelling tasks clearly in order to be able to use this type of task adequately in a technical environment.

"GRUNDVORSTELLUNGEN' AS A THEORETICAL CRITERION FOR DESIGNING MODELING TASKS

In the first section, we took a deeper look on the transition between real situations and the mathematical level. Mental objects are necessary for those transitions as they mediate between reality and mathematics. Referring to this theoretical framework it is the aim of this section to deal with the question how 'Grundvorstellungen' can be used as a normative criterion for creating modeling tasks.

Firstly, we take a closer look at proportions, which are the mathematical concept of the following considerations. The importance of this mathematical content can be seen in the preparation of a simple mathematical model to mathematise many application-related situations. According to Kirsch (1969, 2002), proportions are only understood, if they are apprehended as a transformation which maintain the structure between two quantities. Thereby a transformation f between two quantities G1 and G2 is called proportional, if there is for any $n \in \mathrm{IN}$ and any $a \in G1$: $f(n \cdot a) = n \cdot f(a)$.

In ideal cases, these kinds of transformations between reality and mathematics are based on two 'Grundvorstellungen' (cf. Malle, 2000):

- A connection between quantities can be described, founded or researched. Thereby, the elements of one quantity are in relation to elements of another quantity.
- The effect of the variation of elements of one quantity in relation to the elements of the other quantity is described or observed.

In Figure 2 we can see a simple example for a situation with a proportional context. The modelling structure of this item is the basis for the cognitive demand in these kinds of situations. It is due to this, that we can define the first cognitive level: (1) Relations between different quantities must be identified (on a "horizontal perspective"), i.e. the element of quantity G1 is in relation to the element of quantity G2. (2) The quantities are varying due to the characteristics of proportional transformations (on a "vertical perspective").

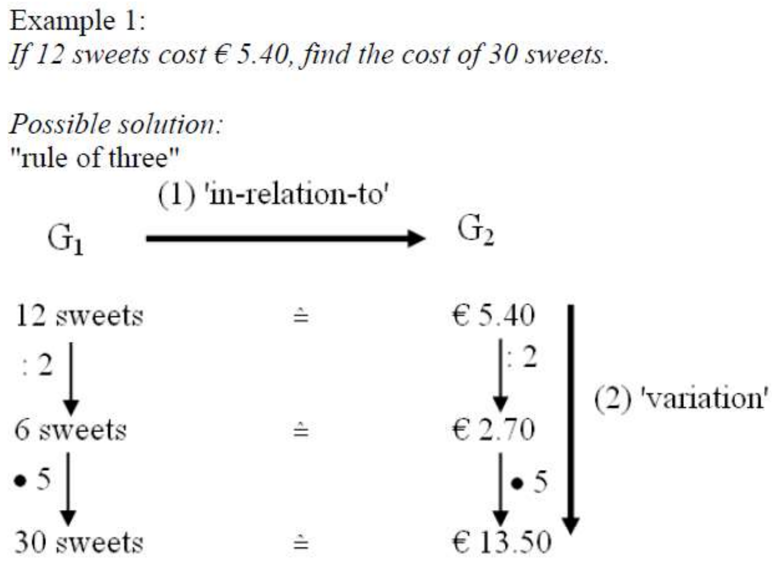

Figure 2: Identification of 'Grundvorstellungen' in an item on a basic level by the "rule of three" between two quantities G_1 and G_2. (Kleine, Jordan & Harvey, 2005, p. 234).

The 'Grundvorstellungen' can not only be integrated such as in Figure 2's solution; it is also possible to activate the 'Grundvorstellungen' for example by a factor of proportionality, by an equality of ratio or by an equality of quotient.

According to Griesel (1997), if we take a look at the demand in percentage calculations on this cognitive level, we can understand percentage quotations as special kinds of quantities. Percentages can be understood as a unit. Particularly with regard to percentage as a special case of fractions, we have obtained the following Grundvorstellungen' for percentage calculations (cf. Blum, vom Hofe, Jordan & Kleine, 2004):

- GV-Percentage 1. The whole is divided into one hundred equal-sized sections. For example, 43% of the students live in the town near to the school. By connecting the 'Grundvorstellungen' with an operation we talk about the hundredth-operator-GV.

- GV-Percentage 2. A statistical point of view whereby a basic set is divided into subsets with the cardinal number of 100. For example, 43% of the students live in the town near to the school means: for every 100 students of the school, 43 students live in the same town as the school.

- GV-Percentage 3. As mentioned above, percentages can be understood as a unit. Therefore, the unit and the value as a whole are a special kind of quantity.

Connected with these specific 'Grundvorstellungen' the basic items for percentage calculations can be solved similarly to the mentioned solution processes above. We can increase the requirement of items by adding further 'Grundvorstellungen' to the former demand in the previous level. This addition is not a trivial combination of 'Grundvorstellungen'. Typical items on this level are the combination of percentage calculations with arithmetical ones, such as 'addition' or 'subtraction' (cf. Padberg & Wartha, 2017). Figure 3 shows a possible solution for items using the process of the basic level above, whereby an additional structure of subtraction is needed.

An additional increase in requirement can be achieved by repeated combination of these aspects. For example, in compound interest calculations or the use of factors of growth in percentage calculations 'Grundvorstellungen' from the previous level have to be arranged in a non-trivial way. In practical work one can expect different kinds of concrete solution processes analogue to the former levels. However, from a theoretical point of view, we can describe the requirement as the interlinking of 'Grundvorstellungen'. The repetition in this level can be combined several times (e.g. within the compound interest calculation). This requirement can have an influence on the used solution method: Even with items which have multiple linking processes the use of operators is superior to other methods because of clarity and cognitive economy.

With regard to the design of modelling tasks as a technical training like in the ASYMPTOTE-project, the requirements arise that one takes into account these different steps of thinking, which can be seen in this example. These thinking steps have an influence on (1) the design of hints and (2) in the structuring of solutions as well as (3) in the structure of task sequences. Especially with regard to task batteries, it has proven helpful that tasks and the individual requirements are built up step by step so that the diagnosis of competences can be made unambiguously. A series of tasks in which example 2 follows tasks on simple percentage calculation and tasks like example 1 give clearer indications of pupils' abilities than example 2 alone would.

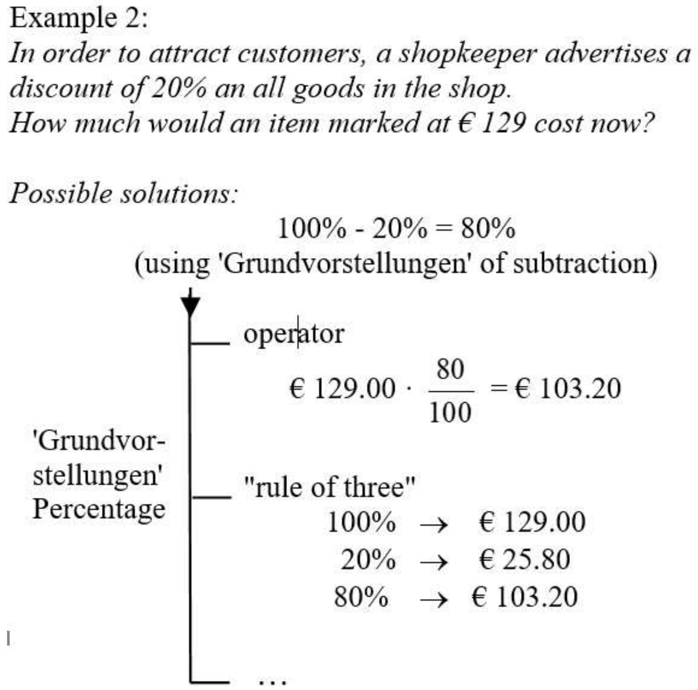

Figure 3: Identification of 'Grundvorstellungen' in an item with higher demand (Kleine, Jordan & Harvey, 2005, p. 236).

SUMMARY

This paper especially focussed on the transition between real situations and the mathematical level whilst working with modeling tasks. For these transitions mental objects are necessary which mediate between reality and mathematics. To be more precise we use the term 'Grundvorstellungen'. 'Grundvorstellungen' can be described as mental models for mathematical objects. We have shown that it is possible to use this concept as a theoretical criterion in order to describe items according to the extent of the 'Grundvorstellungen'. We have used the field of proportions and percentage calculations to illustrate our thoughts and ideas. We have developed in this field a (hierarchical) structure of three levels under the perspective of modeling tasks, whereby the demands on each level are determined by the extent of the used 'Grundvorstellungen'. For the design of modelling tasks, these requirements are an important guideline for the technical implementation. In order to implement competences and skills especially in a digital medium, one needs the form of didactic analysis presented here for the design of task series, solution instructions and sample solutions.

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ENACTIVE LEARNING IN MATHEMATICS AT HOME - THEORETICAL FRAMEWORK

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Abstract. The project "Enactive Learning in Mathematics at Home (EnLeMaH)" aims to promote enactive work of pupils in the area of functional relationships. This paper establishes the theoretical foundations with respect to an understanding of 'enactive learning', learning fundamental, and experimental work in mathematics. This paper is thus the theoretical basis for a workshop on enactive working.

Key words: Bruner, enactive work, experiments.

INTRODUCTION

Introducing an enactive approach to Mathematics teaching helps pupils build a mental network to understand mathematical concepts and relations and how they can use mathematics in their daily lives. Enactive learning means having handmade activities, experiments and concrete handling with material to enter new mathematical topics, have mental representations of mathematical content and discover mathematical relations. Consequently, enactive methodologies help to increase the understanding and the attractiveness of mathematics and, to a broader extent, contribute to reduce underperformance. Nevertheless, the adoption of an enactive approach to mathematics is based on two main preconditions or premises: On the one hand, teachers need to acquire and be equipped with the adequate pedagogical skills to implement this methodology, particularly when it concerns to its applicability to the context of digital education and training. On the other hand, enactive materials can be hard to obtain in the current context. The project "Enactive Learning in Mathematics at Home (EnLeMaH)" promote the adoption of innovative digital pedagogical competencies for mathematics school teachers, which will enable them to develop the knowledge and skills to: (1) Implement an enactive teaching & learning methodology adapted to the context of digital education; (2) Guide pupils in creating, using household supplies, enactive materials that support their learning processes, with a special focus on Mathematics learning in the field of functions.

THEORY OF COGNITIVE GROWTH BY JEROME S. BRUNER

It is fruitful, I think, to distinguish three systems of processing information by which human beings construct models of their word: through action, through imagery, and through language (Bruner 1966, p.1).

As Bruner stated above, individuals represent their learning and the world in which they live through action if they cannot do so using images or words. He assumed about learning that any subject could be taught at any stage of development in such a way that it's cognitive abilities were met. To learn a more highly skilled activity, it has to „be decomposed into

Kleine, M., & van Randenborgh, C. (2023). Enactive Learning in Mathematics at Home - Theoretical Framework. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 41–47). WTM. https://doi.org/10.37626/GA9783959872522.0.05

simpler components, each of which can be carried out by a less skilled operator“ (Bruner 1966, p. 2). Representations are the product of a system of coding and processing past experiences. Hence, he introduced a model including three modes of representation as included in Table 1). He believed that people represent their knowledge in those three ways. Modes of representations “are not structures, but rather involve different forms of cognitive processing” (Schunk 2012, p.457).

<table>
<tr><th colspan="2">Name of the mode of representation</th><th>Description</th><th>Examples in mathematics classes</th></tr>
<tr><td colspan="2">Enactive mode of representation</td><td>Suggests that individuals represent their learning and the world in which they live through action A pupil best understands their environment by interacting with the objects around him</td><td>
Using material to represent a mathematical concept.</td></tr>
<tr><td colspan="2">Iconic mode of representation</td><td>Summarizes events of precepts and of images, by the spatial, tempo-ral, and qualitative structures of the perceptual field and their transformed images.</td><td>Using images (e.g. pictures of the (mathematical) situation, graphs) to represent a mathematical concept</td></tr>
<tr><td rowspan="2">Symbolic mode of represent-tation</td><td>Verbal-symbolic</td><td>Each word has a fixed relation to something it represents</td><td>Using (actually spoken) word to represent a mathematical concept</td></tr>
<tr><td>Non-verbal-symbolic</td><td>Each symbol has a fixed relation to something it represents</td><td>Using written sentences and mathematical symbols (e.g. equations) to represent a mathematical concept</td></tr>
</table>

Table 1: Modes of representation.

For enactive learning, these modes of representation correspond in the learning process (Figure 1). Enactive and iconic representations can yield symbolic representations and vice versa: enactive or iconic representations can be derived from symbolic representations.

By this, the three modes of representation deal with a central theoretical aspect for the EnLeMaH-project: For the understanding and the creation of enactive learning activities the separation between different modes are basic. In the next chapter, we take a deeper look at aspects for designing enactive learning.

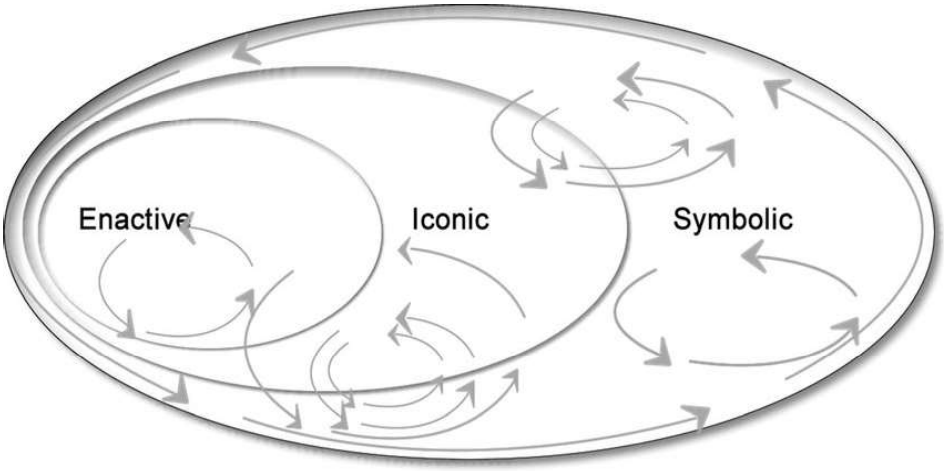

Figure 1: Enactive, iconic and symbolic as nested, co-implicated and simultaneous (Francis, Khan & David, 2016, p. 8).

BIOLOGICAL BASES FOR ENACTIVE LEARNING

According to Di Paolo (2018), the term enactive was used prior to the biological bases that shape the theory today. For example, Bruner (1966) used the term enactive to establish a relationship between representations and bodily aspects belonging to a person's lived experience. Currently the meaning of enactivism is based on the works started by the biologist Francisco Varela and on the works carried out jointly with Maturana (1987). Today this theoretical perspective continues its development by various groups of researchers who are focused on different areas of study (cf. Brown, 2015)

Varela, Thompson and Rosch (1991) used the words 'enaction' and 'enactive' to describe the non-representation list view of cognition they set out, cognition as "embodied action" (Varela et al., 1991, p. 172). This refers to two important points: (1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided. Therefore to understand what enaction is, one must understand what perception is. It is important to note that perception is sometimes seen as a passive process (e.g. when light enters your eyes and you are able to create an image), but in enactivism, perception is an active process, and without action there is no perception. This active process is determined by the structure of the perceiver, for example: how a bird perceives a certain situation is very different from how a person might perceive it. Thus, the organization is understood as the relationships that must exist between the components of something to be recognized as a member of a specific class, and structures of something is understood as the components and relationships that specifically constitute a particular unit that its organization carries out (Lozano, 2014). This occurs in the particular mode of organization which is called autopoiesis. Therefore, an autopoietic system is one that despite being constantly changing and producing new reference systems, the result will always be the same producer. According to Matura (1987), the problem would be how to handle the problem of change of structure and show how an organism that exists in an environment and that operates adequately according to its needs, can undergo continuous structural changes even if the environment is changing. So, this could be an approximation to the problem of teaching mathematics, a learning person is a system that internally organizes itself at every moment. So, every time a stimulus reaches it, (for example a mathematical symbol), it is immediately incorporated into the person's structure, into her being.

According to Lozano (2014), when living beings interact with the environment where other living beings are included and there is a recurring interaction between two systems, then both will change in a similar way. From this perspective, we could say that when a learner interacts repeatedly with her teacher and with the other learners, together they will create a history of interactions. Therefore, the structure of all those who are participating in these classes can change in a similar way, creating new forms of communication and work. If this does not happen, then the structural changes do not lead to adaptation to the environment. Lozano (2014), presents a clear example for this: if a learner repeatedly fails math tests, in a certain context this could mean that the learner changes the learning group he/she is in.

Something important to mention, is that the world is not something that is given to us, but something that we relate to by moving, touching, breathing, and eating, this is what Maturana and Varela called cognition as enactive (Maturana and Varela, 1992). So, enactivism indicates that our mental activity (thoughts, images, emotions) is rooted in the actions we carry out with and through our bodies. The enactivism point of view, learning arises as we actively interact with the environment, so it cannot be thought of as absorption of information and cognition is not a phenomenon that arises within the head or body of a single individual, but arises from continuous interactions with the environment, which in turn is modified by these. In our case, society and culture are part of our environment as human beings.

This concept of enactivism from a biological perspective invites us to reflect on the importance of the type of activities chosen to address a mathematical learning objective. In general, there are many materials available to us, but we need to take into account the context in which our learners are developing, the nature of the environment and the type of structure that makes them up. This means that we must try to create task models that are appropriate to the level of ourlearners and at the same time use materials that allow them to use enactive actions to capture new learning.

EXPERIMENTS AS PART OF ENACTIVE WORK

An experiment is a scientific method designed to gather information. It is used both at school and at university and also in several subjects. Experiments in mathematics education are used in different contexts, especially the difference between experiments in mathematics and other subjects is emphasised (Artigue & Blomhøj, 2013). For our approach here, we focus in particular on the enactive aspects in mathematical experiments. For this purpose, the goals of mathematical experimentation are first explained and the individual steps are derived in the second step.

The method "experiment" – differencies between subjects

Following Kirchner et al., there are different purposes for experiments in natural sciences: gathering knowledge, demonstration of phenomena, giving 'primary experiences' to pupils or the verification of a relation or model (cp. Kircher, Häußler &Girwidz, 2009). All these purposes lead to a better understanding of nature. Typically, there are up to six steps for an experiment in natural sciences. At first the object of investigation must be clarified. Afterwards pupils have to collect hypotheses as a second step. The third and fourth steps are the planning and execution of the experiment. Whilst execution the measurement of data is important in order to analyze these data for correlation between quantities. This analysis is the fifth step and is only followed by the last step: the interpretation of results. In the last step the results and hypotheses are compared (cp. loc. cit.). The interpretation of results itself often leads to another object of investigation and thus to another experiment. Even though there are differentiations between experiments (e. g., will pupils or the teacher execute? or in what phase of the lesson is the experiment integrated?) in natural sciences every experiment is about real objects.

There are similarities and differences between mathematical experiments and experiments in natural sciences. In both subjects an experiment describes a way of gathering knowledge by observation of controlled action with 'objects' (cp. Ludwig & Oldenburg, 2007, p. 4). The process of experimenting in mathematics is largely identical with the process in natural sciences. Though it is not necessary to collect hypotheses before trial. Examining several examples or handling with material is a starting point for pupils to build hypotheses, so step two can be replaced after step three and four. As mathematical facts need to be proved the sixth step of interpretation suggests approaches to a formal proof or leads to a repetition of the experiment with slightly different conditions (cp. Philipp, 2013, Goy & Kleine, 2015). Finally mathematical experiments can be detached from real objects. Thus, experimenting in mathematics needs the pupil to know heuristics and teaches process-oriented competences.

Mathematical experiments as a process

As mentioned before, experimenting in mathematics is a cycle of different steps. Referring to Philipp (2012) or Goy and Kleine (2015) there are four main steps:

- Stating the mathematical problem/question
- Generation of hypotheses
- Planning, execution and analysis of the experiment (short: 'trial')
- Elaboration of a mathematical model, concept or proof

For every experiment stating the mathematical problem or question is the first and the elaboration of a model, concept or proof is the last step. The order of the other steps can be changed considering the experiment's aim. If the experiment is about to verify or falsify hypotheses, those hypotheses have to be generated first. If the experiment aims on pupils learning how to experiment or making up their own models and concepts, the trial has to be placed before the generation of hypotheses (cp. Goy & Kleine, 2015, p. 5f).

Heintz constructs three contexts for mathematical experiments: discovery, validation and persuasion (cp. Philipp, 2013, p. 25). Context of discovery pertains to the generation of hypotheses and is meant as systematic trial in order to explore unidentified relations. Here knowledge is obtained by induction. Otherwise, knowledge is obtained by deduction when a given hypothesis is validated by the mathematical experiment. In this case the experiment is set in the context of validation. Finally, if neither discovery nor validation is needed because a relation, concept or model is already confirmed there is another context for mathematical experiment: persuasion. In this case the experiment shall convince the pupils (cp. loc. cit.).

Based on the theoretical background, enactive learning at the EnLeMaH-project can be described as hand-based activities, which enables pupils to discover mathematical relations or prove mathematical connections. The different phases of an experiment can be a guideline for teachers on the basis of the designing principles, to arrange an enactive learning situation.

SUMMARY

In this paper, an understanding of enactive learning has been laid that starts from the historical roots of Bruner and also focuses on the biological aspects of an active learning. The understanding of enactive learning should be concretized by looking at mathematical experimentation and its conditions. The understanding of enactive learning will be concretised by looking at mathematical experimentation and its conditions. Mathematical experiments find different approaches, this contribution was about the enactive approach. The explanations are intended to integrate the EnLeMaH project, to which this article refers. In this project, a training programme for teachers was developed to enable this enactive work synchronously or also asynchronously, even when learning at a distance. More information and access to the project can be found at www.enlemah.eu.

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BRINGING 3D PRINTING INTO STUDENT TEACHERS' MATHEMATICS EDUCATION

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Abstract. 3D printing has been used in classes by teachers previously in several studies. In Germany, a lot of schools lack the funds or opportunities to use 3D printing in classes on a broad scale. Therefore, a seminar is in development and in the second iteration at Goethe University to train student teachers (educators) to create their own manipulatives for mathematics classes, where they worked intensely on mathematical and didactical aspects of their manipulatives. We present the underlying theory of 3D printing, and an ongoing teaching experience with student teachers using 3D printing for mathematics education. Additionally, students' expectations were evaluated to improve future iterations of the seminar. Some manipulatives from the seminar are presented.

Key words: 3D printing, manipulatives, student teachers.

INTRODUCTION: 3D PRINTING IN MATHEMATICS CLASSES

There are certain topics in education that benefit greatly from examples that can be represented by three-dimensional objects, like the cubic formulae (Figure 1) in mathematics or sugar molecules in chemistry. This is especially interesting for natural sciences, where the physical world, real objects, schematic models, miniatures and similar can be scaled up or down and be held in one's hands, observed and investigated. These materials are called manipulatives (Larbi & Mavis, 2016). But not every object or example can be bought (for a reasonable price) or manufactured for a whole classroom without significant strain and effort by teachers and educators.

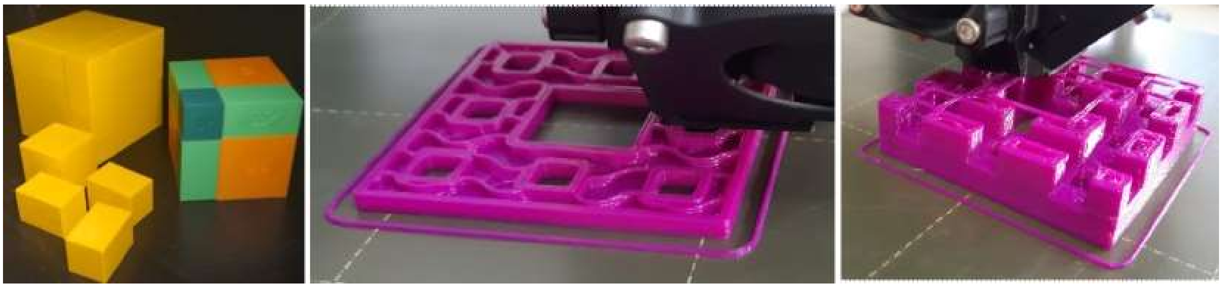

Figure 1: A student's manipulative (left), and the process of additive printing (center and right).

3D printing (3DP) is a practical way of manufacturing three-dimensional objects. Even though there are variations on how exactly, the most common one is additive manufacturing. A digital 3D model is sliced by a software into layers that then subsequently get printed with molten plastic onto a build plate, where every layer provides the needed support for the layers above. This technology was used first in an industrial context, where prototypes of certain objects could be manufactured quickly. Through the years 3DP has become more accessible to the public due to reasonable prices and is therefore now available to use in schools and education in general. (Gür, 2015)

Läufer, T., & Ludwig, M. (2023). Bringing 3D Printing into Student Teachers' Mathematics Education. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 49–56). WTM. https://doi.org/10.37626/GA9783959872522.0.06

It has been shown that integrating 3DP in STEM education impacts mathematical knowledge, spatial ability and technical skills, as well as attitudes towards mathematics, engagement and motivation positively (Kit Ng, Tsui, & Yuen, 2022). Pearson and Dubé (2022) found that 3DP is used to connect different subjects in K-12 education and engage students with challenging topics, but mainly when students have the chance given by their educators, school and curricula to use 3DP during lessons (Anđić et al., 2022; Ford & Minshall, 2019; Kit Ng et al., 2022; Pearson & Dubé, 2022). A lot of schools lack the technical requirements to make this happen: Be it missing funds or the high maintenance needs of printers; 3D printers may be difficult to obtain or keep intact and ready for printing (Bull et al., 2015); German schools' technical infrastructure and their progress lag behind the current needs for digital education (Lorenz et al., 2021) and there are several problems when using 3D Printers in schools, such as long print times, high maintenance needs, and high instructor effort, and low access to help when facing difficulties as an educator (Anđić et al., 2022; Bull et al., 2015; Ford & Minshall, 2019; Pearson & Dubé, 2022). Some teachers also feel that 3DP is not ready to be used in schools due to the aforementioned problems. (Ford & Minshall, 2019)

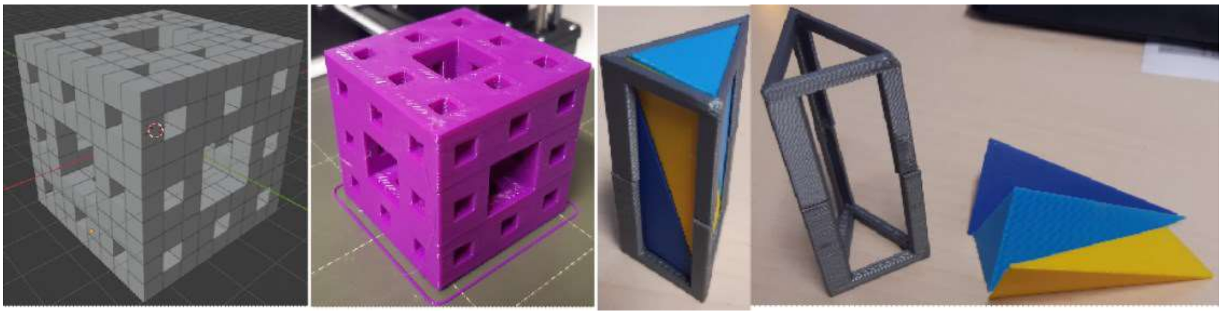

Figure 2: A 3D model of a Menger sponge (left), the model printed (2nd to left), and a manipulative of a prism that is "cut" into three equivalent pyramids (right).

Due to this a focus on using 3DP as an educator to manufacture manipulatives that aid learning is appropriate. This brings the benefit of reduced instructor effort during lessons, reduced cost due to fewer printers and materials needed, and less need for maintenance. For this, a seminar has been developed to teach educators (student teachers) how to use 3DP to enrich mathematical lessons, following Ford and Minshall (2019)'s identified need to include 3DP in pre-service teaching to allow teachers to use 3DP in their lessons. „The issue 'teaching the teachers' is a pressing one but one that appears to have been overlooked" (Ford & Minshall, 2019, p. 156). Teaching educators about 3DP and using 3DP to produce artefacts that aid learning are typical uses of 3DP in the education system (Ford & Minshall, 2019).

Anđić et al. (2022) have found that teachers may also see 3D-Modeling-Printing (3DMP) workflow as a trigger for themselves to design new tasks for students, new activities for their classroom, and "modernizing and adapting approaches and teaching methods" (Anđić et al., p. 8). In addition, it may be "a useful teaching tool for their professional development, as it increases their abilities to teach STEM together with their digital, communication, and evaluation skills" (Anđić et al., p. 12).

With 3DP, realizing mathematical objects is "easier than ever" (Asempapa & Love, 2021, p. 87). Objects like the "Integraph", a historic mathematical device that creates an antiderivative of a function graphically (Dilling, 2020), can be manufactured in a cost-efficient - and once familiar: comfortable - way, to allow a historical approach to integration in calculus. Other examples include edge models of special solids (Hoffart, 2019), and specific support materials for special needs students (Kalina, 2019). In one case abstract mathematical models such as a Möbius strip have been fabricated (Gür, 2015) to allow the physical investigation of abstract objects. In an example case presented in our seminar, we show a prism that has been sliced into three equivalent pyramids (2), allowing an enactive introduction to the volume of a pyramid. Fractals may also be addressed via a Menger sponge, which is an example used (Figure 2).

Through this and other materials, students approach the relations between three-dimensional bodies in an embodied, enactive, hands-on way (Bruner, 1966; Tall, 2013). 3DP, as an example, "can (also) be used as the basis for students' mathematical modelling experiences" (Asempapa & Love, 2021, p. 86) when creating models reflecting the real world. The use of 3DP in schools may also provide an effective method for exploring modeling in a more realistic context; and can be used as the basis for students' mathematical modeling experiences (Asempapa & Love, 2021)

A lot of models can be downloaded from a publicly available website like "Thingiverse" and then printed on the available printer in the school, although there is a lack of high-quality model databases for education - specifically for mathematics (Ford & Minshall, 2019). To not have educators rely on models that are not tailored for education, and to allow educators to innovate their educational materials, teaching them how to design and print their models is appropriate.

THE STRUCTURE OF THE SEMINAR AND EXPERIENCES MADE

Following this approach, we have developed a practical, weekly seminar (2 x 45 min) at Goethe University where student teachers get introduced 3DP and develop their own didactic materials. The course spans a whole semester (12 weeks) and has been completed once. The experience was used to adapt the course for a second implementation, following a basic approach inspired by the design-based research approach (Cobb et al., 2003; Cobb et al., 2016; Seel, 2012).

Phase	Duration	Content
Input	5 weeks	In this Input-phase students get introductions to the topics of 3DP, the 3DMP workflow, didactics of physical models as well as the technical capabilities of the scripting library, mainly Basic 3D Bodies, Translations, Algorithms & Iteration, as well as constructive solid geometry - the way to create new objects by using Boolean operators like difference, intersect and union on two existing objects.
Hands-on	5 weeks	During the Hands-on-Phase the students begin and continue working on their models with minimal intervention from the educators. Through this, the students have the freedom to design their materials and realize their ideas. Access to the 3D printer is available during and outside of the course hours, during which

		upcoming problems are addressed and models are discussed in one-on-one discussions.
Presen-tation	2 weeks	The presentation phase, only consisting of two weeks, served as a frame for the student teachers to present their materials to their peers, answering questions and explaining the theoretical use in school. The didactic materials must be embedded in theoretical but realistic lessons and the state curriculum for mathematics.

Table 1: Phases, their duration, and their content of the seminar at Goethe University on 3DP

The first time the seminar was held we developed a prototype version of a mathematical scripting library in the modelling and animation program Blender, which allows the students to use the programming language Python to be used to construct 3D models. Over the first semester, we saw that the students had trouble realizing what a 3D printer can be used for, which we have traced back to not letting the students get first-hand experience themselves very early on. This problem especially led to several situations where students explained an idea for a manipulative that wasn't printable in one part or it may have been difficult to realize with an additive 3D printer's restrictions, for example, overhangs where the printer would've needed to print in the air, or large objects which would have taken multiple days to print. They had a lot of ideas but the knowledge about the restrictions of a 3D printer wasn't known to them. Additionally, students had trouble starting with the three dimensions and designing three-dimensional objects. We also continuously added new features and worked on enhancing the scripting library to have more features such as more complicated objects like a wire frame or hollow cone.

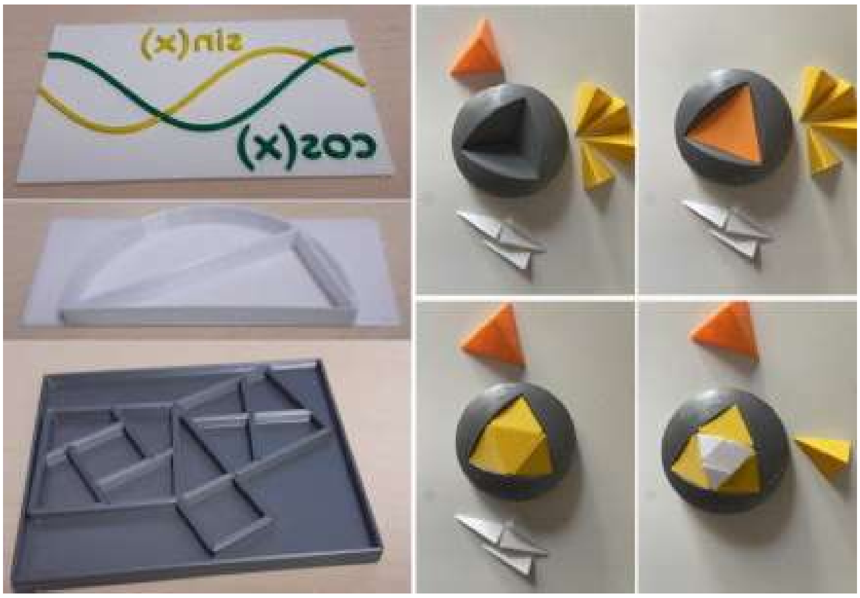

Figure 3: The "cookie stamps" example (left; sine/cosine, Thales' Theorem and Pythagoras' Theorem), and a students' manipulative (right) for introducing the approximation of a sphere. A "hole" in a sphere can be filled with pyramids whose top edges have the length of the circle's radius. Using more pyramids (yellow and white) approximates the sphere better than less (orange).

In the second iteration of the seminar, following the experiences of the first iteration, we started by reducing the number of dimensions the students worry about and began printing students' creations in the second session. The task was to design a cookie stamp with mathematical concepts or proofs (see Figure 3). There, students need to design a cookie stamp with 2D proofs or mathematical principles (i.e., Geometric series, Pythagoras' theorem, Thales' theorem, see Figure 3) that they are familiar with and whose individual components only feature circles, lines and points. These were printed in the second session and the students were very motivated. They could take their stamps home with them. The reasoning behind these manipulatives has been explained to the students and used to explain what a printer can and cannot print, after which students' ideas for manipulatives included the 3D printer's restrictions in the design itself. Explaining programming structures as if-else and loops have also helped students formulate their ideas more freely. The second iteration is being held in the winter semester of 2022/2023 until March 2023.

STUDENTS EXPECTATIONS

To gain insight on the student teachers' needs and wants regarding this new technology we pose the question *What expectations do student teachers have towards a 3D printing seminar in mathematics education?* The answer to this question may lead to further insight on how a seminar to 3D printing may be designed to raise student motivation and gain knowledge on student needs towards such a seminar.

To answer this question there were two questionnaires via SosciSurvey that the students answered voluntarily. Through a shared, anonymous code in both questionnaires that can be recreated by knowledge only the students have, their before- and after answers could be combined. There they could react to their initial expectations and whether they were fulfilled.

In the first questionnaire, 11 students (3f, 6m, 1d), on average in the 9th semester (10 answers, mean = 9.1, std=1.37), aged 23-27 (10 answers, mean: 24,4, std: 1,17) reported on their previous knowledge regarding 3D printing. The second questionnaire was filled out by 10 of the 11 students.

Only one student had previously 3D printed their design, and one other had watched a video or read something about it. The students have not used any products commonly used in 3D design or printing (Blender, TinkerCad/AutoCad, SketchUp, Ultimaker Cura, PrusaSlicer, Paint 3D, POVRay, Maya, Fusion 360), except one who had experience with Blender and another who had experience who another, unmentioned program. The student teachers were asked to freely formulate their expectations towards a 3D-printing seminar ("What are your expectations of the seminar?"). These answers have been used to inductively create categories, the occurrences of which are shown in Figure 4. In the second questionnaire, their initial expectations given in the first questionnaire were given back to them with the question of assessing whether they were fulfilled. Only 6 students answered this item, with 5 of them agreeing that their expectations were fulfilled, and the last one only agreeing that it was instructive.

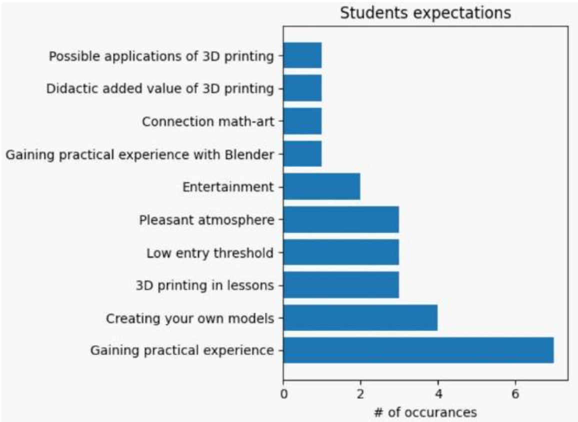

Figure 4: Students expectation categories and how often they occurred during the questionnaire before the semester began (left), and Students answers on 3D printing in math education (right, excerpt).

STUDENTS MANIPULATIVES FROM THE FIRST SEMESTER

In this chapter, we highlight two didactical materials created by students. First, we have an approximation of a sphere (Figure 3), where a part of a sphere has been cut out and is replaced by a pyramid (orange) whose edges have the same length as the sphere's radius. It is directly visible that there is a significant portion of the sphere missing when substituting this part of the sphere with a pyramid. A substitution of the orange pyramid with four yellow pyramids, where each of them again has their edges' length be the spheres' radius shows, that the sphere is now "rounder". There is less of the sphere missing and we are approximating the sphere more appropriately. If we now substitute the center yellow pyramid with four white pyramids we further approximate the sphere. This model especially highlights students' needs for complex modelling, thus our decision to use script-based modelling is justified.

With this material teachers can introduce the concept of limits in a hands-on way. Students can see that by using more pyramids, each has its edge length be the spheres' radius, we approximate the sphere more and more. Of course, at some point, the 3Dmodel is insufficient in approximating further, but the concept can be grasped. An introduction to limits in the other direction - starting with a high number of pyramids approximating the sphere well - is also possible. A different model is shown in Figure 1 of the cubic formulas. The concept itself is not new, but the student has expanded the possibilities by adding colours, engravings, and distractor elements into the models. Through that, a larger differentiation can be achieved, as well as aid for special needs students through the engravings which can be felt by touch.

For higher-achieving students there is the puzzle with distractors, through which they need to figure out which elements to choose and which are not included in the formulas' results. It also lacks any kind of distinguishable color or engraving. Lower-achieving students can be aided by the color, engravings, and by taking away the distractors.

DISCUSSION

The enactive, hands-on approach to student teachers designing and printing their manipulatives seems to be a very motivating experience, which was expected when comparing it to constructionism, where a construct serves as a representation of lessons learned (Papert, 1980). But due to the low number of students, a statistically significant effect has not been shown.

Using a script-based creation of 3D models may not be suited for everyone. We experienced that lower-achieving students struggle with algorithmic structures and basic expressions. A mix of both may be suitable depending on the students' needs and maybe even their orientation (higher, lower, special education). If they focus on less technically complex and more didactically complex objects, options like TinkerCad are very well suited to create didactically rich 3D models. In a hands-on way, students can deepen their knowledge in didactics regardless of their orientation (primary, upper/lower secondary, special needs) but applying it to their model is motivating and may lead to long-lasting learning. This has been shown in other research too (Ford & Minshall, 2019).

The freedom of the students to create what they want - within reason - has produced interesting and thoughtful models as well as models where mathematical content is hardly found. A constant discussion about what the goal is: that real students may use this, "so what should they do and learn?" helps guide the students to focus on the necessary.

We come to the same conclusions and experiences supported by various research (Ford & Minshall, 2019; Kit Ng et al., 2022; Pearson & Dubé, 2022) that 3DP is a valuable tool that can be used in education and may even help teachers further develop their professional acting (Anđić et al., 2022). We especially want to highlight the points that schools may lack the funds, teachers may lack the time to fully use 3DP in-lesson, or maintenance may be too high, which is why a focus on educators creating their manipulatives via 3DMP is very applicable and, proven by the presented seminar, possible even in the beginning of teacher education, namely universities.

Further questions may be what types of models may be created by either a script-based or a graphical way of creating 3D models and what the pros and cons are, or how student teachers' didactical and mathematical is fostered using 3DMP.

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TOPIC 2

INNOVATIVE APPROACHES OF USING DIGITAL TECHNOLOGY IN EDUCATION

TECHNOLOGY SUPPORTED ACTIVE LEARNING

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Abstract. Active learning promotes student-centered approaches to the learning process, allowing students to develop skills and competences that traditional, passive learning methods cannot foster. In turn, supporting active learning with digital technology tools creates new possibilities in terms of pedagogical design and implementation. This article shows how active pedagogical methodologies like problem-based learning, design thinking, and others can be effectively supported by digital environments and tools like collaboration platforms, serious games and virtual and augmented reality by presenting several projects that I've been involved in the past few years. Therefore, it is also an overview of my recent research and practitioner activity in that domain.

Key words: Active Learning, learning technology, serious games.

INTRODUCTION

Learning is a personal process that is influenced by a learner's existing knowledge, abilities, skills, motivations, and other factors. The traditional, industrial approach to education that assumes all learners will respond identically to the same teaching methods has proven to be ineffective. This passive approach fails to tap into a student's individual experiences and doesn't help them develop the skills needed for their future careers.

Active Learning is a pedagogical approach that encourages students to take an active role in their own learning. By involving students in their education, active learning recognizes and leverages their personal experiences and helps them reach higher cognitive levels. The concept of Active Learning was popularized by Bonwell and Eison (1991), who defined it precisely as a method where students actively participate in the learning process. They emphasized that students must do more than just listen to lectures, they must also engage in activities like reading, writing, discussing, and problem-solving. This approach aligns with the constructivist ideas that students build their understanding by making connections between new information and their previous knowledge. Active learning improves critical thinking, problem-solving, motivation, collaboration, communication, entrepreneurship, and integration into society. It also fosters lifelong learning by promoting student autonomy and control over their own education.

Active learning can be implemented through different methodologies like problem-based learning, project-based learning, experiential learning, design thinking, inquiry-based learning, and others. And while these methods often involve group work, they can also be used for individual reflection.

Many studies have shown that active learning is also efficient, with students in active learning courses being less likely to fail and performing better on exams and assessments than students in passive lecture-style courses (Vaz de Carvalho & Bauters, 2021).

Active learning places a higher degree of responsibility on the learner, but instructor's guidance is still crucial to direct learners to the right path. Skilled teachers and facilitators

Vaz de Carvalho, C. (2023). Technology Supported Active Learning. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 59–68). WTM. https://doi.org/10.37626/GA9783959872522.0.07

promote learning by providing challenges, advice and support adjusted to the student’s ability. Teachers usually find that the increased level of academic discussion with their students is much more rewarding than the simple passive lecturing.

Active learning can be facilitated through technology, such as virtual learning communities, personalized learning platforms, games, simulations, virtual labs, and virtual/augmented reality systems. These interactive and immersive tools, which are considered advanced Open Educational Resources (OER), can be used to effectively support higher education students in developing required competencies. Policymakers, researchers, developers, and educators must take these emerging technologies into account and design educational programs that utilize these new-generation learning tools to enhance the learner's autonomy, collaboration, creativity, and critical analysis skills. Learning with these tools should prioritize visual, auditory, tactile, experiential, and interpretive experiences to facilitate active knowledge construction. (Batista et al, 2008; Pereira et al., 2007).

In this article, we describe a set of projects that show how active learning can be applied to different target groups, using different pedagogical models and technologies.

PROJECTS AND BEST PRACTICES

The following set of exemplary active learning implementations derive from different projects applied in different European countries and result from the last years of research and practice of teams that I’ve collaborated with in this domain.

ALIEN (Active Learning in Engineering Education) project

The ALIEN project introduced an active learning intervention for engineering education which gathered 4 European universities and 12 South Asian Universities. The project setup problem-based learning environments addressing real-life issues related to science, technology, engineering, and math (STEM) concepts. The intervention aimed to facilitate the more effective transition of students from the academic environment to the world of work through digital activities linked to engineering curricula skills mismatches through learning scenarios inspired by real life industry and societal challenges (Tsalapatas et al, 2021; Tsalapatas et al, 2019).

Figure 1: PBL Lab at University Tenaga Nacional, Malaysia.

The project also promoted the deployment of digital technology as a complementary learning medium that enriches student engagement and provides meaningful feedback. The project's intervention was based on a complementary set of vectors:

● Problem-Based Learning (PBL) laboratories: The ALIEN consortium established physical PBL laboratories in 12 universities in South Asian countries including Malaysia, Vietnam, Cambodia, Pakistan, and Nepal (as shown in Figure 1). The labs were customized to meet the specific needs of each institution and enabled active learning through the integration of digital services and educational content. The labs facilitated the use of digital tools such as serious games, simulations, and augmented/virtual reality in educational settings through specialized equipment. These labs also had writable surfaces for group work on solution design and 3D printers for prototyping, providing a comprehensive and technologically advanced learning environment.

● Digital Platform for Engineering Education: The digital services were created to enhance educational activities through increased engagement, interaction, and feedback. The platform allowed educators to structure, publish, and reuse problems, challenges, and activities, providing access to rich, digitally-enhanced educational content and facilitating student interaction across participating universities.

● Community Services: The ALIEN platform had a strong emphasis on community building to encourage knowledge exchange among educators, experts, industry players, and students regarding best practices in problem-based learning in engineering education. The services were open to all interested in problem-based learning and included features such as profile management, messaging, activity tracking, and forums. The special interest groups aimed to further promote problem-based learning through discussions on topics such as gamification, AI in education, tools and approaches, specific areas like software engineering, and best practices.

● Instructor Training: The ALIEN program aimed to address the issue of insufficient instructor training, which was identified as a hindrance to modernizing higher education. The instructor capacity building services aimed to empower educational institutions to effectively integrate digitally-enabled problem-based learning into their existing practices to better prepare students for the workforce.

LEAP (Lean and Agile Practices Linking Engineering Higher Education to Industry) project

The LEAP project aimed to educate engineering students in higher education on emerging lean and agile design industry practices to better prepare them for the workforce. The project showed how these practices can be applied to a variety of engineering contexts beyond their original sectors, such as automobile construction and software engineering, to improve production efficiency, reduce waste, and prioritize customer needs through user-centered design (Rodriguez et al., 2018).

The project utilized active learning through hands-on experience in a virtual environment that mimicked real-world industry practices in the form of serious games. The first game demonstrated the benefits of the 5S model of lean practices in diverse sectors and the second game highlighted the value of the SCRUM Agile Design Model in engineering for both end-users and producers (see Figure 2).

Figure 2: The LEAP serious game.

NATURE project

The NATURE project is a European initiative that creates a highly interactive digital learning platform to educate higher education students on responsible natural resources management and environmental sustainability. The platform raises awareness of the significance of sustainable natural resources management and provides students with both theoretical knowledge and practical skills to become active citizens in designing sustainable solutions. NATURE utilizes a digital game that simulates real-life scenarios where students act as stewards of natural ecosystems, balancing preservation, economics, and quality of life. The game provides instant feedback on the consequences of their choices, enhancing their knowledge on the impact of human activity on the environment (Caeiro-Rodriguez et al, 2022).

The NATURE project's educational framework is based on the results of a learning needs analysis and takes into account existing environmental education practices and policies. The platform also offers educational activities inspired by real-world challenges and can be easily adapted to meet different needs. These activities aim to build knowledge, skills, and positive attitudes towards responsible natural resources management through a game-based approach.

AUDID (Adults, Data, and Emerging Identities) project

The AUDID project was established to address the growing issue of digital illiteracy among the adult population, particularly those aged 55 and above, who are at risk of social exclusion and vulnerability with regards to the use of digital tools and internet safety. The goal was to enhance their awareness of online identities by providing practical, motivating, and easily understandable training and learning tools that would protect them against dangers that come with online presence and cybercrime. The project aimed to enhance the quality of life, social participation, and cohesion of these individuals, increasing their online confidence and trust, and thus fighting against senior exclusion (Zanchetta et al, 2022).

The consortium developed an innovative, experiential pedagogical approach and a set of interactive digital learning tools to improve understanding of the subject and related competencies, such as critical thinking. These tools were designed to provide recognition of skills and achievements through Open Badges.

To meet the needs of the adult population, the solution was implemented in three parts. The first two involved creating an interactive multimedia curriculum for adults and adult educators to empower them with essential skills and knowledge, and providing tools to understand the risks associated with online identities. The third part involved providing practical exercises for adult educators to facilitate the training themselves and become key facilitators.

The solution was interactive and experience-based, designed to engage learners, link to the growing importance of visual information, and have a dual implementation capability, both in the classroom and online. To apply the acquired knowledge in real-life situations, the project developed a dynamic demonstrator, an interactive web-based tool based on real-life scenarios, available in all national languages and including 28 different scenarios. This tool served as a valuable methodological tool for adult learners to empower their transversal skills and develop their professional competences. All of these features were integrated into a Learning Motivation Environment (LME) that supported social and peer learning, the interactive multimedia curriculum, and the dynamic demonstrator.

MINDLIVEN (Mindfulness in Nature) project

The MINDLIVEN project is focused on using mindfulness-based training to help individuals cultivate mindfulness and integrate it into their daily lives. Mindfulness can impact the six main areas of work design - demand, control, support, relationship, role, and change - by altering automatic thought processes, increasing response flexibility, and promoting working memory, self-determination, and persistence. This can lead to better problem-solving, decision-making, and attentional control, as well as higher creativity and empathy (Menardo et al, 2022).

Mindfulness also promotes cognitive flexibility and adaptive capacity, allowing individuals to better adapt to new situations and think outside of past patterns. Exposure to nature can also positively impact stress and mental fatigue, reduce negative moods, enhance positive emotions, and provide calming effects. The MINDLIVEN project has developed a mindfulness-based instructional approach that combines formal training with mindfulness practices and exposure to visual and immersive nature environments (Figure 3).

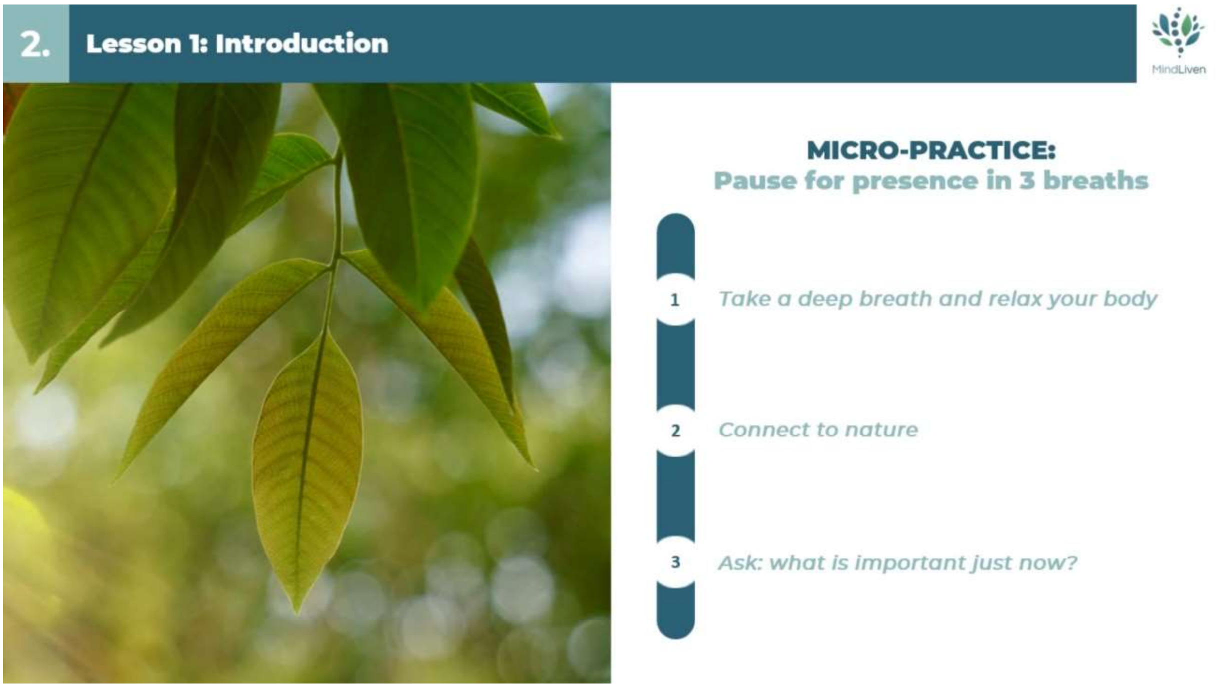

Figure 3: MINDLIVEN practices.

DeSTRESS (Developing Competencies for Stress Resilience @SMES) project

The project focused on creating a digital platform for enhancing workers' and managers' skills in handling technostress at the workplace. The centerpiece of the project was the design and creation of a digital game-based training platform, which was developed based on the findings from the initial phase of the project. The game utilized a 3D role-playing approach, providing learners with a dynamic, immersive, and hands-on experience in dealing with stress caused by technology in the workplace. The role-playing aspect was also reflected in the character attributes related to technostress, whose values changed based on the player's progress, tracked through game analytics, to evaluate the player's performance and learning outcomes. The development of the platform followed an Agile software process, with regular interaction with end-users. Both employees and employers were the target audience for the game, which was designed to train both groups on how to handle technostress (Pasini et al, 2021).

Before playing the game, learners completed a self-assessment questionnaire to determine their level of technostress. During the game, they would alternate between the role of employer and employee and face various scenarios where they would need to choose the best course of action. They could respond through language, actions, or postures, and the consequences of their choices would determine whether they gained or lost points based on their ability to manage technostress effectively (Figure 4).

Figure 4: The DeSTRESS game.

HERA (Higher Education Re-engineering through Active Learning for Growth) project

HERA developed a digital learning game platform that aimed to challenge students by integrating knowledge from multiple fields and simulate the way engineers and economists work and collaborate in real-world scenarios. The platform is set in a virtual city, where players can take on various roles representing different stakeholders and make decisions related to city design, budget allocation, and citizen satisfaction, among others. The game allows players to collaborate and requires them to balance various interests and resources (Caeiro-Rodrígues et al, 2020).

The platform features a game editor with various options for designing complex scenarios, including terrain formatting, a wide range of buildings and city infrastructures, and other elements such as parking lots and city decorations. It also implements traffic, weather, day/night cycles, and seasons, all of which affect the scenario's evolution as the game is played. The game provides a rich and immersive experience, allowing students to experience the complexities of real-world scenarios (Figure 5).

Figure 5: The HERA serious game.

LoEL (League of Emotions Learners) project

The project aimed to empower young people to enhance their emotional intelligence, allowing them to recognize and express their own emotions, and to establish effective communication both online and offline, especially in professional settings. The project had five main objectives: to create empowering training materials that help young people understand the source and nature of emotions, to offer a diverse range of activities that mix real digital communication methods and environments, to provide linguistic expressions that express basic emotions across different cultures, to teach appropriate verbal and nonverbal cues to communicate effectively in negotiations and conflict situations, and to offer a motivating training approach to organizations that work with young people (Santos et al, 2021).

The target group was young people and youth trainers who could benefit from the results by becoming more aware of their emotional intelligence, recognizing the benefits of managing emotions, and knowing how to express emotions and communicate through digital means. They could also learn how new technologies and their own channels and signs can shape communication and express emotions.

The LoEL approach brought together two important aspects that are not often combined in a training process: the significance of language in identifying and expressing emotions and the creation of professional scenarios through hands-on learning and gamification strategies suited for the business world.

The LoEL app uses innovative gamified educational methods, tailored to the digital habits of young people, to help players develop their emotional intelligence. This combination of digital content in the form of a serious game available on a mobile platform has been shown to be highly effective for the target group, as measured by their ability to apply the learned skills in real-life tasks. The app includes three key components: "Emotions Box," a multilingual dictionary of emotions; "Express Yourself," a collection of games to test and improve emotional expression skills; and "Emotional Organizations," an activity that allows players to role-play and practice managing emotions in professional scenarios. The app includes a progression system that moves players through increasing levels of difficulty.

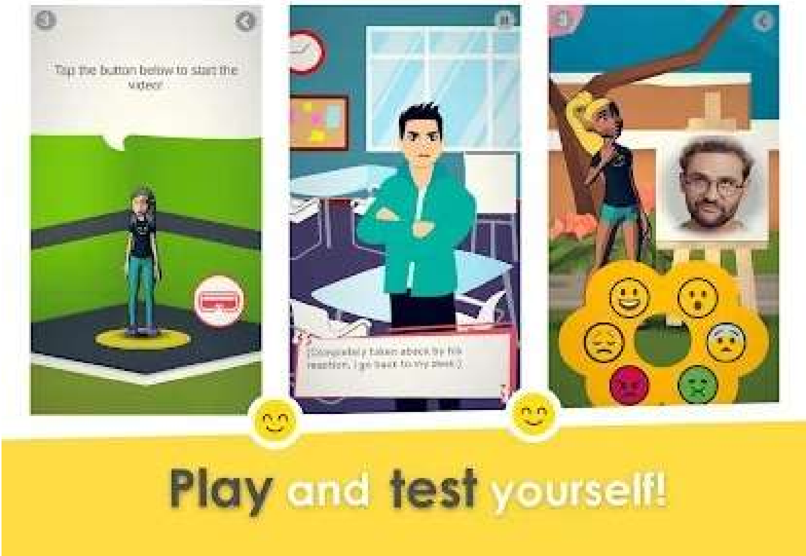

Figure 6: The LoEL serious game.

CONCLUSIONS

Active Learning is a pedagogical approach that motivates students to take an active role in their learning process. This approach leverages their personal experiences, encourages higher cognitive levels and improves improved critical thinking, problem-solving, motivation, collaboration and communication. Active learning can be implemented through different pedagogical methodologies like problem- and project-based learning, experiential learning, design thinking, and inquiry-based learning. The use of technology allows to use an extended range of pedagogical activities and methods to support active learning practices.

As a researcher and practitioner (teacher and trainer), Active Learning has been a major focus but also tool in the past few years. This article intention was precisely to show that Active Learning can be effectively and successfully deployed using different pedagogical approaches and technologies and for different target groups.

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GEOMETRY MAPPING TOOL: IMPROVEMENTS ON A DESCRIPTIVE GEOMETRY LEARNING TOOL

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Abstract. *Over the past decades, the technological presence has been increasing in education, producing new applications and digital platforms to assist with learning. This technological shift raises the achievement of students and promotes better school performance. This paper aims to explore the improvements made on Geometry Mapping Tool, a web application to support the teaching of Descriptive Geometry, specifically, the Monge method. The improvements were result of gathering criticism from students and professors in the field of study. The decisions made to improve the tool can provide useful insights to other educational tools. The web app was then re-evaluated to capture the feedback on the changes made and additional improvements.*

Key words: Descriptive geometry, technology education, web development.

INTRODUCTION

In the last decade, technology and the digital world have shown exponential growth and, consequently, technological solutions have emerged in various fields of study. The teaching of Descriptive Geometry (DG) is no exception and has undergone changes with the introduction of new technologies in the classroom, both on a physical level (e.g., computers and projectors) and digital level (e.g., PowerPoint presentations and virtual school platforms).

Stachel (2003) summarized DG as a method to study 3D geometry through 2D images providing insight into structure and metrical properties of spatial objects, processes and principles. The education in Descriptive Geometry provides a training of the students' intellectual capability of space perception.

"At present, the basic content of DG is taught in the last years of pre-university education and in practically all branches of Engineering; it is of vital importance in Design, Mechanical and Civil Engineering." (García et. al., 2007)

The purpose of this article is to discuss the improvements made on a DG learning tool specifically designed to assist the teaching of Gaspard Monge's method (Barbosa & Pereira, 2022).

Monge's method resorts to Orthographic Projection (OP) to describe the 3D geometry on a sheet of paper. The 3D geometry is projected to two perpendicular planes called: Vertical Plane (VP) [φ] and Horizontal Plane (HP) [υ] (Figure 1). The line of intersection of these two planes is called the Ground Line (GL) [x]. To represent the system in the 2D model, the HP is rotated around the GL until it coincides with the VP (Müller, 2022).

The developed learning tool is called Geometry Mapping Tool (GMT). While the traditional classroom studies of DG relies on paper sheets to faithfully represent a 2D plane, this tool comprises a web application that allows users to observe and interact with geometric representations on both 2D and 3D systems simultaneously. The tool was developed and

Barbosa, J. R., & Pereira, J. P. (2023). Geometry Mapping Tool: Improvements on a Descriptive Geometry Learning Tool. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 69–76). WTM. https://doi.org/10.37626/GA9783959872522.0.08

implemented by the first author with guidance and supervision from the second author of the paper.

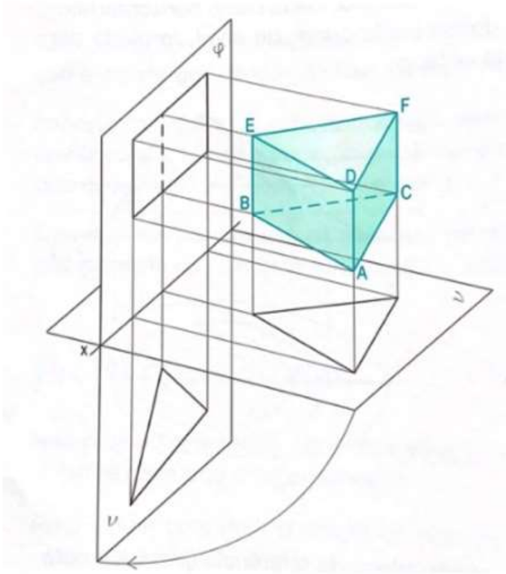

Figure 1: Monge's method (Müller, 2022).

The current paper is divided in six sections, being this one the first and introductory section. Following this, section 2 is reserved to discuss the related work and the technological decisions made to develop the application. Section 3 documents the engineering process followed in the development of the tool and its functionalities. Section 4 focuses on the implementation of the improvements and new functionalities and section 5 covers the results of a usability evaluation on the new features. The article ends with a small and concise conclusion about the work and prospects for the future development of the tool.

BACKGROUND

Throughout the years, there have been many attempts to introduce software in the teaching of DG and each approach has advantages and disadvantages. Currently, DG is lectured in a specific subject integrated in the Science and Technology Course and Visual Arts Course for the high school education. Since the main target for this tool was pre-university students and teachers, the criteria used to evaluate the effectiveness of the tools was based on three principles (Barbosa & Pereira, 2022):

- **Accessibility:** Accessible and compatible with most devices regularly used by DG students and teachers.
- **Dynamism:** Dynamic in the visualization of 3D space so that the user can have the best spatial perception of the exposed geometric elements.
- **Functionality:** Describes as many features as possible to demonstrate the content of the DG field of study.

We can group the technological learning solutions into two types: web platforms and executable applications.

Web platforms such as GeoGebra (cf. Hohenwarter & Jones, 2007) offer easy access and compatibility since they are mostly free web platforms requiring only a browser to access. Most platforms address the complete GD curriculum, thus demonstrating a strong functional aspect. However, these web platforms perform poorly according to the dynamism principle and the best that they can offer is a step-by-step view of DG exercises and their respective 3D view.

Executable applications are more diversified in terms of functionalities and capabilities and most of them offer a good dynamic presentation of the 3D geometry. AutoCAD has been a choice in many university classes and has gathered interest from students to learn DG. However, for a pre-university class, the software is costly, and the level of complexity brought to the user can become a big obstacle to the learning experience (Bokan et al., 2009).

Another candidate is the application “AEIOU – Geometria Descritiva” that focuses on teaching the Monge method (cf. Alves, 2008). Developed in 2001, the executable is not compatible with the modern Operating Systems (OS) and the Graphical User Interface (GUI) looks outdated. Nonetheless, the tool is simple and clear for teaching DG and more specifically the Monge method.

The Geometry Mapping Tool (GMT) seeks the combination of these three approaches, taking advantage of the strengths identified and reducing the problems pointed out in each of them. Therefore, the tool was designed and developed to become a highly accessible web platform with the functionalities and dynamic visualization from the executable applications.

To display 3D graphics in a web environment, the tool was developed with a JavaScript framework named ThreeJS. This framework is built around the Web Graphics Language (WebGL) library and provides a level of abstraction that facilitates the development of the tool without compromising it. To support this framework the tool was implemented with a simple web stack: Hypertext Markup Language (HTML), Cascading Style Sheet (CSS) and JavaScript (JS)/ TypeScript (TS). These technologies allow the development of web platforms where HTML establishes the structure of the page, CSS describes the style of the page and JS/TS defines the logic and functionality of the page (ThreeJS, n.d.).

As of now, the platform migrated to a modern web framework called SvelteKit. This decision was made due to the increasing work in development and the need to be up to date with the current web technologies. SvelteKit is a full-stack meta-framework (a framework built on top of another framework) that ties the frontend and backend together to deliver the best developer and user experience. This framework allows the developer to build high-performance web apps with ease due to the Hot Module Replacement (HMR), where changes to the code are reflected in the browser instantly. SvelteKit provides features like preloading pages and offline support to facilitate the accessibility from low-performance devices. With this framework, the project can be implemented at a higher pace and deliver new functionalities without compromising the user experience. Being a full-stack framework, SvelteKit opens the possibility of implementing a backend that integrates with a database to persist data within the two representational domains thus, expanding the use cases of the tool (Joy of Code, 2022).

GEOMTERY MAPPING TOOL

The development of the tool followed every step of the software engineering process with the gathering of the functional and non-functional requirements, modeling the system at different levels of abstraction and describing the solution components. To understand the core concepts and relationships between the components of the application, Figure 2 exposes a simplified version of the domain model of the system.

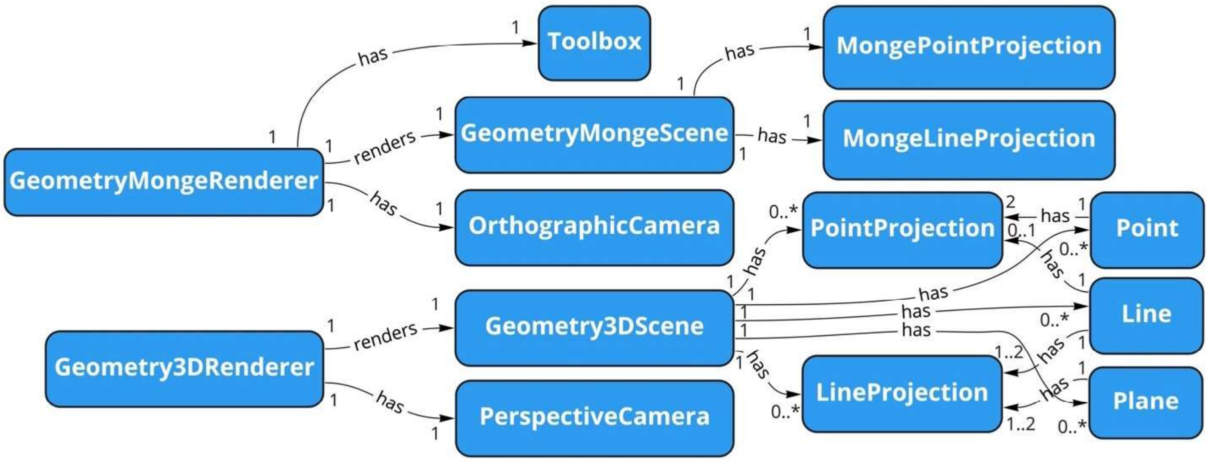

Figure 2: Simplified domain model of the application (Barbosa & Pereira, 2022).

The application is divided into two domains: the 3D domain and the Monge domain. These two domains characterize, respectively, the real-world 3D view and the 2D representation with the help of the Monge method. Consequently, the Graphical User Interface (GUI) is split between two windows representing the respective domain. In Figure 3, we can observe these two domains highlighted with a color (highlighted in yellow for the 3D domain and in blue for the Monge domain). To help with the creation of the geometry in the Monge view, the application provides a set of tools in a toolbox component (highlighted in red).

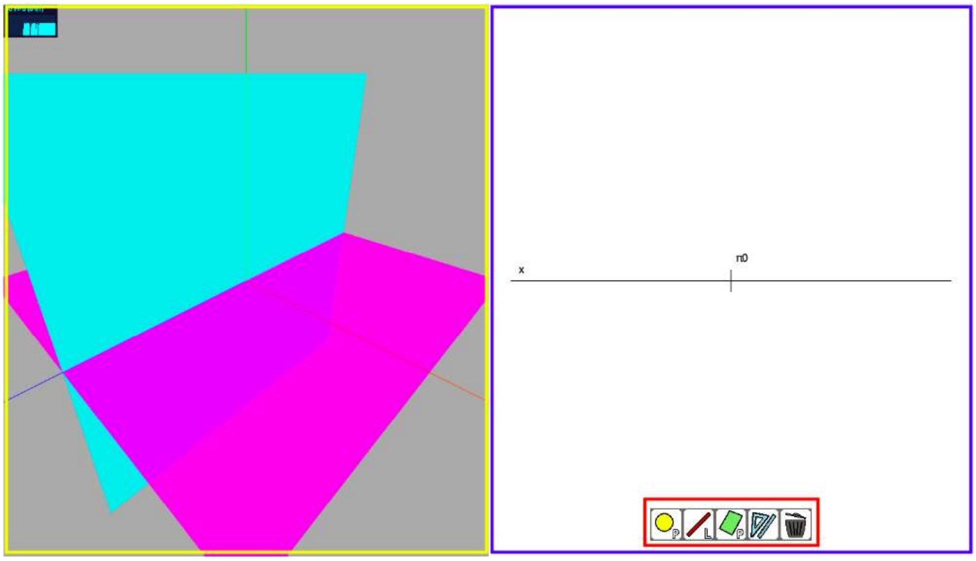

Figure 3: GMT's GUI (Barbosa & Pereira, 2022).

This way, the application supports the creation of geometric elements in the Monge view, simulating the experience of the user solving exercises in the DG class and simultaneously providing the real-world representation in one single screen.

GMT'S IMROVEMENTS IMPLEMENTION

The major criticisms to the previous version of the tool targeted the use of the point tool. The problem users had when dealing with this tool was that it provided poor accuracy since it allowed the creation of a point projection where the cursor was positioned. An improvement to fix this low accuracy problem was to fix the selection of the y coordinate along the x-axis and when chosen, fix the x coordinate and wait for the selection of the y coordinate. By splitting the selection process of the coordinates, it allowed the user an easier control and more accuracy when creating the point projections.

Another criticism focused on the GUI for being outdated and hard to grasp. After taking these insights into account, the toolbox was updated to a much simpler GUI creating space for new tools. Since the appearance of the toolbox was also a target of criticism the icons were updated to give a modern style. Figure 4 represents the previous version of the toolbox's GUI on the left and the recent design on the right. For a better comparison both tools have all their buttons hovered to demonstrate all changes made. The older version of the toolbox was composed of five major tools: point tool (a) allowing the creation of point projections, line tool (b) allowing the creation of line projections, plane tool (c) allowing the creation of plane projections, auxiliary line tool (d) allowing the creation of line or plane projections perpendicular or parallel to an existing one and the delete tool (e) allowing the removal of any projection. Point, line and plane tools had options to change between projection types ("F" for frontal projections and "H" for horizontal projections).

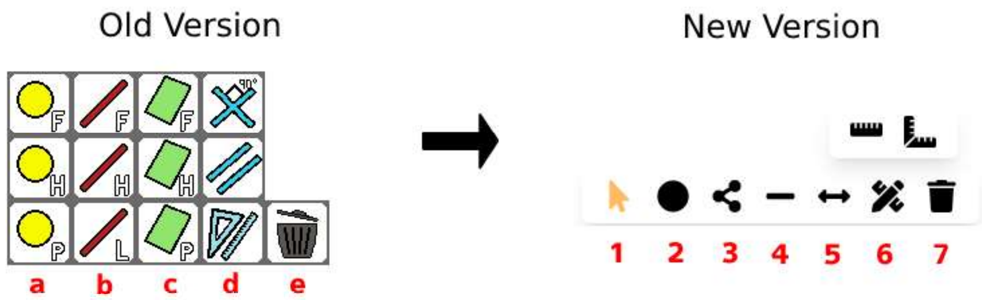

Figure 4: Previous and current version of the toolbox's GUI.

In the Monge method the type of projection is defined by the number after the name of the projection (e.g., "a1" represents the horizontal projection of the line "a"). It was concluded that there was no reason to have separate buttons for the creation of each projection type and instead move that decision to the prompt after the selection. Furthermore, in the Monge method a point projection can represent part of a line (Figure 5) and a line projection can also represent plane projections. Therefore, the line tool and plane tool become redundant, and the point tool can be modified to be compatible with the line projection creation leaving these decisions to the prompts after selection. The current toolbox's GUI kept the four tools: point tool (2), line tool (4), auxiliary line tool (6), delete tool (7) and displayed three new tools that we are going to cover in this section: view tool (1), point association tool (3) and segment tool (5).

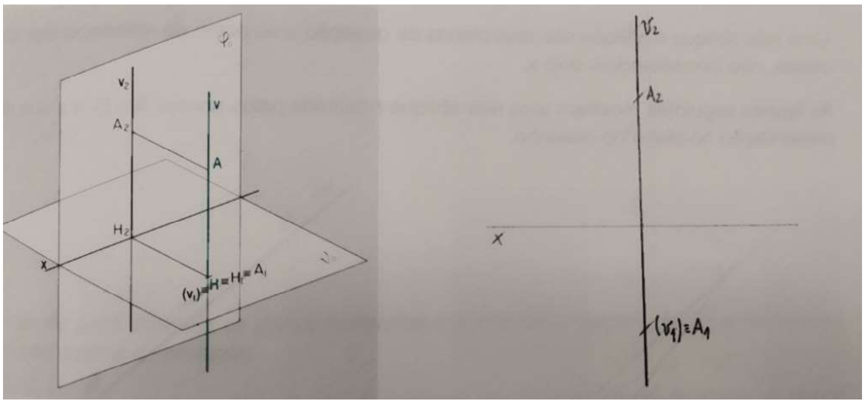

Figure 5: Vertical line composed of a point projection (v1) and a line projection (v2) (Müller, 2022).

User experience (UX) is a key factor when designing the tool and the purpose of the Monge view is to simulate the work environment of a DG's student/professor. The point association tool allows the user to select a line within the Monge view and create a point projection of that line along the x-axis. This feature facilitates the user when creating point projections where a line projection intersects another.

Another tool that enhances the UX is the view tool. This tool allows the user, to a certain degree, zoom and pan the Monge window. By having this available, the user can guarantee a more rigorous use of the tool and, consequently, have more accuracy when using the other tools. Along with this feature, it was implemented an auxiliary button to set the zoom and position of the camera back to their default values.

Currently the application is in a work in progress state, so it doesn't cover the complete DG's curriculum. To help accomplish that, the segment tool was implemented. This tool allows the user to connect point projections hence creating segment projections and 3D segments. This feature contributes for the completion of the DG's curriculum by enabling the construction of 3D figures and solids (Figure 6).

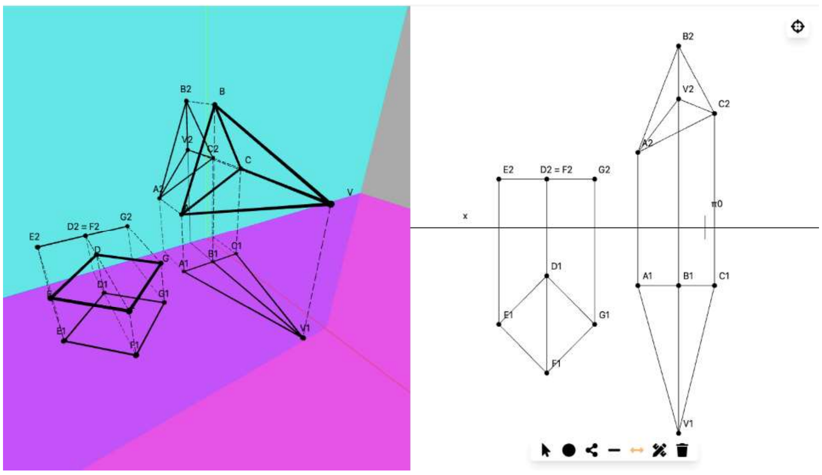

Figure 6: Square and a pyramid built with the segment tool.

EVALUATION

To properly assess the implementation of the new improvements an evaluation session was conducted. In this evaluation, the participants solved a DG exercise, adjusted to the tool's capability. Upon conclusion, the participants gave an informal opinion about the tool, on a practical level, by addressing three topics: criticisms of the system and aspects for improvement; benefits of having the 3D representation view; suggestions for additional functionalities. The session occurred on the 7th of December 2022 and there was a total of four participants: three students and one tutor of DG from the tutoring center "Grupo Peculiar". Participants had no previous experience with the tool at any stage of development.

At the start of the activity, participants pointed out that the selection of point projections from the use of the point association tool was sometimes difficult to make. However, with the help of the zoom and pan from the view tool this difficulty nearly disappeared. One participant noted that a substantial amount of time was wasted switching tools and so, suggested the use of the view tool simultaneously with the other tools via keyboard shortcuts.

When asked about the 3D representation view, all participants agreed that the tool contributes greatly to the understanding of the Monge method and improve the intellectual capability of space perception. Furthermore, the 3D view provided additional information about the exercise and gave the ability to check for mistakes and rectify them.

Participants made a lot of suggestions regarding new functionalities for the tool. Most suggestions were minor improvements to the existing tools such as: highlighting point projections that have the same x coordinate; have the point association tool work for segment projections; facilitating the creation of point projections in line projection intersections. Another great suggestion that resonates with the tool's objectives was the ability to save and load exercises developed with the tool.

CONCLUSION

Following the opinions from the evaluation session, the tool seems to be at a very optimistic state and the improvements implemented have been proven helpful and positive. The 3D view continues to serve an important role in developing the user's space perception and it was identified a desire for UX improvements in this view. As stated in section 2 and reported in section 5, the meta-framework SvelteKit improves the development rate of future improvements for the tool and opens the possibility of implementing complex use cases.

Continuously capturing feedback from the academic world is a vital step in the software engineering process of the tool. To become successful in the education environment, it's recommended for the tool to be put into practice on a larger population with an extended timeframe.

A reminder that GMT's purpose is to assist students and professors in the teaching and learning of the Monge method and not substitute the traditional approaches used in DG. Manual practice is of great importance for success in DG and the rigor required in it is part of the criteria used to assess students. In this way, the desire remains for the tool to be further developed and applied in classrooms, thus fulfilling its purpose.

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AN EXPERIENCE WITH AUGMENTED MATH TRAILS AND SERVICE-LEARNING IN INITIAL TEACHER TRAINING

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Abstract. *We present the ongoing project developed at the Universidad Autónoma de Madrid (Spain) using math trails in Initial Teacher Training with the aim that they can incorporate this educational resource into their future professional career. The learning experience is an integrated project focused on the creation of math trails by prospective students as a valuable outdoor problem-posing activity. In addition, it includes a two-step evaluation process that, first, allows prospective teachers to assess each other's routes, and secondly, they are tested by students as a service-learning activity. The math trails used in the project include mobile technology as an enriching element, combining the experience of using MathCityMap and the Augmented Reality tools of GeoGebra within the same project.*

Key words: Augmented reality, initial teacher training, math trails.

INTRODUCTION

Math trails can be defined as "a walk to discover mathematics" that can be almost everywhere (at the street, inside buildings, in parks, museums, zoos, etc), with marked stops "where walkers formulate, discuss, and solve interesting mathematical problems" (Shoaf et al., 2004). They are very flexible and creative educational resources. They can take place in urban, natural, or artistic settings, and can be adapted not only to any educational level, but also for tourism or divulgation purposes.

It is a form of outdoor learning which connects mathematics inside and outside the classroom. Students must solve mathematical tasks along the route, so it is a problem-solving activity in a real context which highlights the presence and relevance of mathematics in common objects and daily situations. It also gives them the opportunity to have an active and collaborative mathematical learning experience in a more relaxing atmosphere. Math trails are meant to use the spaces around us as an educational space, so they have a great potential as an interdisciplinary tool. Therefore, the inclusion of math presents as an opportunity to develop their mathematical competences and to provide future teachers with an experience of creative and collaborative work related to mathematics that they will be able to incorporate in their professional future (Barbosa & Vale, 2016; Moffett, 2011).

This paper presents a learning experience at Universidad Autónoma de Madrid with pre-service teachers where they use and design math trails with mobile technologies. The main goal is to involve prospective teachers in an integrated project with the aim that they can incorporate this educational resource into their future professional career. In addition, it gives students an opportunity to provide a service to the society with their learning in an authentic setting.

For the appropriate integration of technology in educational activities, the theoretical framework that stands up is the TPACK (Technological Pedagogical Content Knowledge)

Benito, A., Nolla, Á, Gómezescobar, A., Sánchez, E., & Ajenjo, C. (2023). An Experience with Augmented Math Trails and Service-Learning in Initial Teacher Training. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 77–84). WTM. https://doi.org/10.37626/GA9783959872522.0.09

developed by Mishra and Koehler (2006). There have been several previous integrated approaches to enhance pre-service teachers' TPACK such as the SQD model in (Tondeur et al., 2020). In this direction, the present paper follows a teacher training strategy divided in four stages: (1) experience, (2) design, (3) peer-review, and (4) service-learning.

This experimental inclusion of innovative tools in Mathematics Education belongs to an ongoing research project. During all the stages, we have asked our students to fill out questionnaires, quantitative and qualitative, before and after the experience. In addition, they hand in the group dossiers with the problem-solving strategies.

THEORETICAL FRAMEWORK

Math trails in initial teacher training

The experimentation and creation of math trails is an enriching activity for prospective teachers (Barbosa & Vale, 2020; Haas et al., 2021; Martínez-Jiménez et al., 2022b). It is presented as an open challenge, becoming a mathematical creative learning process which requires the design of contextualized tasks for their future pupils. It provides them with a practical activity which shifts from usual academic settings to real-life mathematical situations. It constitutes an opportunity to develop problem-solving and problem-posing skills in prospective teachers, and to provide them with a collaborative mathematical experience that they can incorporate in their professional future (Figure 1).

Figure 1: Prospective students at UAM in a math trail (left) and creating a mathematical task (right).

The aim is to engage them with this outdoor learning process and give them the confidence to include math trails in their professional future. It is a collaborative math experience new for most of them, learning how to look at our surroundings with "mathematical eyes".

Use of mobile technology in mathematical education

The use of mobile technology has been successfully implemented in several outdoor math learning proposals. It gives more flexibility and autonomy during the activity and gives teachers the possibility to give feedback and interact with students (Cahyono & Ludwig, 2019; Wijers et al., 2010). In relation to math trails, in this paper we consider two: MathCityMap and the Augmented Reality (AR) tools of GeoGebra.

MathCityMap

Developed at the Goethe Universität Frankfurt in 2012, it is an easy-to-use online platform (https://mathcitymap.eu/) and a mobile App where teachers can create, share, and carry out math trails. Some of the key features that makes it a useful tool to perform and create math trails are the possibility to create *digital classrooms* to carry out trails with students, the GPS geolocation of the trail, the interaction between students and teachers via chat, the inclusion of gamification features, hints to solve the problems and immediate feedback after the task is concluded (Ludwig & Jablonski, 2021).

Augmented Reality

One of the applications of AR tools into educational settings is its introduction in outdoor activities such as math trails. This technology adds an extra “mathematical layer” to the location or to the real object considered in a task, increasing the variety of possible math problems and ways to solve them. Works in this direction can be found in Botana et al. (2020) with GeoGebra AR and Cahyono et al. (2020) with Secondary Education students (Figure 2).

Figure 2: Uses of Augmented Reality in Botana et al. (2020) (left) and Cahyono et al. (2020) (right).

THE MATH TRAILS PROJECT AT UNIVERSIDAD AUTÓNOMA DE MADRID (UAM)

Ongoing since the academic course 2019-2020, we have been using math trails at courses of *Mathematics and its didactics* at undergraduate and master’s degrees in education at UAM. The degrees involved are the Early Childhood Education Degree (3-6 years old), Primary Education Degree (6-12 years old), Master in Secondary Education (12-18 years old) and Master in Innovation in Specific Didactics. In total, more than 600 prospective teachers have participated in the different stages of the project.

The main purpose of this project is the creation of mathematical routes by the future teachers (Stage 2). For this, a previous step of experiencing a mathematical route is necessary (Stage 1), which is done using MathCityMap. To complete the learning process, the trails created are tested and self-assessed by the students in a peer-review session (Stage 3). With the feedback provided, the math trails are redesigned and finally implemented with school students (Stage 4). Details about the 4 stages are the following:

- **Stage 1: Math trail experience**. Most of the students had never taken a math trail in their previous education, so the initial step is to experience a mathematical route. The students are divided in groups and join a *digital classroom* in the MathCityMap App to follow the trail. The gamification features are activated to keep track of the scores of the groups at every single task (Figure 3).

Figure 3: Image of the UAM Campus and location of mathematical tasks.

There are 5 pre-designed routes in MathCityMap around the UAM Campus, aimed for the different degrees involved:

Route 1. For students of the Early Childhood Education Degree (MCM code: 564103).

Route 2. For students of the Primary Education Degree (MCM code: 343382).

Route 3. For students of the Primary Education Degree in the optional course *Mathematics in Art and Nature*. The tasks of the trail are based on natural elements of the surroundings, taking students to the neighboring Monte de Valdelatas (MCM code: 127856).

Route 4. For students of the Master in Secondary Education taking the specialty in Mathematics (MCM code: 082917).

Route 5. Augmented Reality route designed for students of the Master in Secondary Education taking the specialty in Mathematics (MCM code: 133601).

In addition to the mobile app, every group has a dossier to write down their problem-solving strategies and calculations. In some tasks, mostly in Routes 1 and 2, students are asked to solve them in the dossier (the task is set up as an *Information Point* in the app). This simulates a task in a math trail without the use of technology, which is relevant for students in the Early Childhood and Primary Education degrees since their future students will not (usually) have access to mobile devices (Figure 4).

Figure 4: Students during a math trail with MathCityMap at the UAM Campus.

- **Stage 2. Creation of math trails.** Prospective teachers design in groups math trails aimed for their future pupils. The work at this stage can follow two options: (1) students create their math trails at any location around the city (streets, nearby schools, etc.), or (2) students create their math trails at the UAM Campus.

Figure 5: Math trail created by a prospective teacher in Madrid.

In Option 1, there is more freedom to choose the location of the route and the relationship between mathematics and day-to-day situations is better solved. In Option 2, the proximity to the University makes easier to review and test the routes created (Figure 5).

The use of MathCityMap is usually optional, although in some cases it is mandatory (for example, when the groups create their trails at the UAM Campus or with students of courses of *Technology in Mathematics Education)*.

- **Stage 3. Peer-review.** When the students create their math trails at the UAM Campus it is possible to do a review session where groups of students (peers) carry out each other's math trails. The MathCityMap portal is very helpful during this process, since it gives the possibility to create groups for students to share and modify their tasks.

During the peer-review session, students evaluate the work of their partners filling out a rubric for every completed task, and a general questionnaire at the end of the trail.

- **Stage 4. Service-learning.** This final stage connects the learning process of the prospective teachers with their professional future and seeks to enrich their creative outdoor proposals with a service to their community. To this end, the trails designed by prospective teachers are tested with either schoolers (if the routes were created next to a school), or students from schools visiting the UAM (if they were located at UAM Campus).

The data collected in Stage 3 together with the teacher's evaluation serve to redesign the previous work and elaborate the final mathematical routes that will be put into practice with school students.

EXPERIMENTING WITH AUGMENTED REALITY

Prospective Secondary School teachers experienced a MathCityMap trail augmented with GeoGebra 3D applets. This experience was carried out by students of *Technology in Mathematics Education,* where they had previous training with GeoGebra. In addition, before doing the augmented math trail they had a pre-session with these tools. They were asked to model different wooden solids with GeoGebra and check if their model fitted the real 3D object (https://www.geogebra.org/m/s2kcquhj) (Figure 6).

Figure 6: Images of the modeling activity with Augmented Reality and GeoGebra.

With this activity students learn how to place and manipulate GeoGebra objects and to obtain the perfect adjustment of the 3D model. To do so, they need to make several test-redesign processes, thus going through modeling cycles during the activity.

Augmented Reality tasks in math trails

The *augmented math trail* requires two mobile apps: MathCityMap and GeoGebra 3D Calculator. Students are divided in groups and enter a digital classroom in MathCityMap where they can follow the route. At every task, there is a link to a GeoGebra applet with augmented data of the location or the object (a 3D model, hints, extra data, etc).

Once the applet is open with GeoGebra 3D Calculator the student can place the model into the reality. The objects in this outdoor setting are bigger than the pre-session models, so the difficulties in the placement of the 3D-models increase. Nevertheless, having the GeoGebra tools at their disposal was of great help for solving the tasks.

Some examples of possible uses of Augmented Reality in the outdoor setting of math trails include: (1) AR as a help for solving a task, (2) the use of AR as a dynamic model or (3) providing new or missing data of the object/location. Examples of this uses contained in the Route 5 in the UAM Campus are the following:

Example 1. AR as a helping device. In this type of task, the GeoGebra applet shows the 3D model of the object related to the mathematical problem. With GeoGebra check boxes, texts, and dynamic tools, like sliders, it is possible to highlight the key parts of the mathematical model or include measures and hints. Figure 7 shows the example of a task asking to calculate the volume of the recycling box, where the 3D model highlights the measures and details of its structure to help students in their calculations.

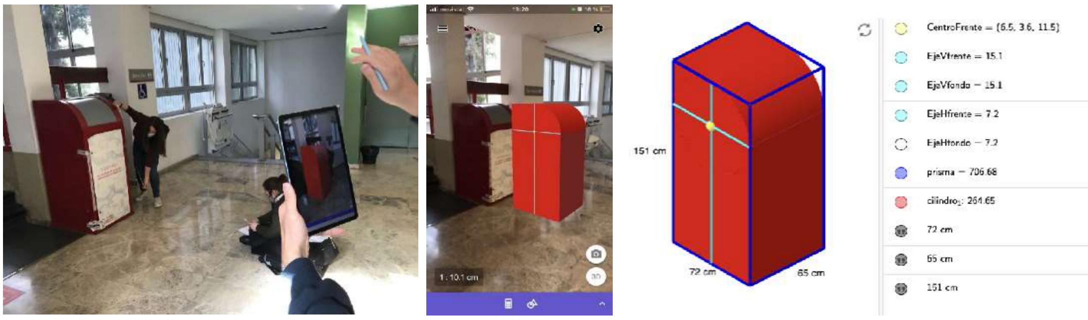

Figure 7: Students using the AR in a math trail and the GeoGebra model.

Example 2. AR as a dynamic model to solve tasks. Students can use the GeoGebra model to solve problems by placing the object in the location and looking at the data that GeoGebra displays. For example, it is possible to check volumes of flowerpots (Figure 8, left) or place stairs in an open space (Figure 8, right). In both cases, with the use of sliders the students can adjust the models to reality.

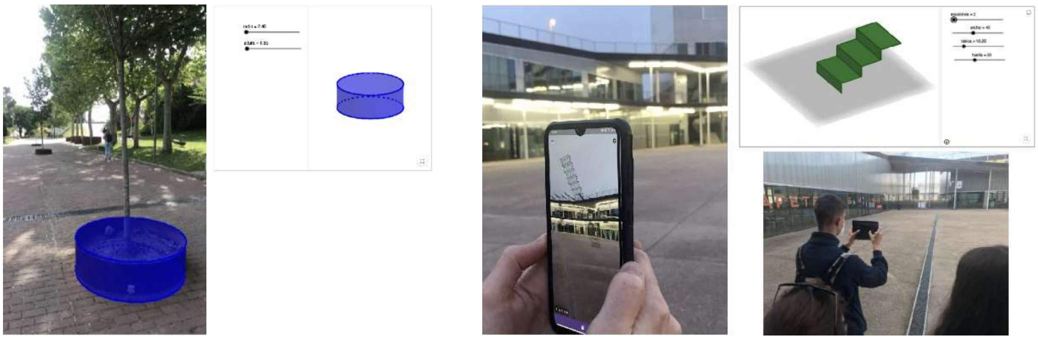

Figure 8: Augmented math trail tasks: *Which flowerpot is bigger?* (left); *How many steps do you need to reach the top of the building?* (right).

Example 3. AR providing new or missing data to solve tasks**.** With Augmented Reality tools it is possible to enrich the mathematical content of the problem by including new or missing objects of the location. This is the case of the task shown in Figure 9, where students can see an extension of the tiles forming the floor of a sculpture at UAM Campus. The goal was to measure the amount of metal plates that the artist bought, before cutting them to make the final rectangle (shown in green).

Figure 9: Augmented task where students have the extended model of the floor.

CONCLUSIONS

Following the evidence given by the questionnaires filled in by students, in general terms, the project has been very positively evaluated. They showed positive attitudes towards math outdoor activities, and most of them found the inclusion of MathCityMap and GeoGebra engaging and motivating. For instance, the SIMS test (Guay et al., 2000) during the math trails showed high scores in the positive dimensions of motivation, while the negative dimension had low scores. Partial results in this line can be found in (Martínez-Jiménez et al., 2022a). Moreover, they considered the creation process of math trails meaningful for their professional future. Indeed, some of them have included math trails in their teaching, creating new routes near their schools.

In relation to the AR experience and its inclusion into math trails, the future Secondary School math teachers showed good responses in participation and motivation, as well as noticing room for improvements concerning the GeoGebra AR tool. For instance, they found it difficult to place the models into real objects of big size, mainly because of perspective misalignments and the objects do not remain fixed at the location.

During the augmented math trail, the use of AR which required more interaction with the GeoGebra calculation tools to solve the task were considered more engaging by the students, finding very attractive the idea of carrying out a math trail with such technology at their disposal. Nevertheless, most of the students appreciated the potential of AR in education but still see the implementation in the mathematics classroom far away, so further experimentation and research in this direction is required.

Finally, in relation to teacher training and the use of mobile technologies, it should be noted that although it provides prospective teachers with technological competence, it comes to limitations when putting into practice with their future pupils. Namely, in terms of access and use of these technologies and some constraints in the types of mathematical problems that can be posed.

Regarding Augmented Reality, the process of experimenting with its inclusion in mathematics activities should continue. Despite the advantages of GeoGebra being a very versatile software, the AR tool needs further development for its use with large objects outdoors. In addition, it would be appropriate to explore other software, as well as the potential of using markers and GPS positioning in math trails.

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VIRTUAL REALITY STEM TRAILS: EXPLORING MATH TRAILS WITH STEM EDUCATION APPROACH IN A VIRTUAL WORLD

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Abstract. This study explores how STEM-based math trails may be conducted virtually. We used the STEM Education approach, math trails concept, and virtual reality technology to develop a town-based STEM learning environment. In this project, a virtual reality mobile app was produced that can be used with Cardboard VR. Teachers build virtual trails with mathematics tasks related to municipal landmarks. Students utilize a virtual reality program on their phones to explore the trail. They employ the mathematical modeling cycle to solve real-world STEM problems. These activities can be done anywhere, anytime using cutting-edge technology. Experiments and deployment in several contexts are needed.

Key words: Math trails, STEM education, virtual reality.

INTRODUCTION

Interdisciplinary STEM curriculum helps students shift from theoretical to practical learning (Bergsten & Frejd, 2019; Corlu et al., 2014; Holmlund et al., 2018; Just & Siller, 2022; Kertil & Gurel, 2016). Teachers that desire to embrace STEM education have several options accessible. When a country succeeds in STEM (science, technology, engineering, and math), its prospects of worldwide success and prosperity grow (STEM). These subjects must be taught to prepare pupils for society's present and future requirements. Math's practical applications may be more visible, but it's the foundation of STEM (Just & Siller, 2022).

The actual world provides STEM classrooms ideas. In many countries across the world, students may learn math while visiting historical and cultural sites. Based on errand data, pathways may be established between different task locations. They may pick a route with math diversions. The "math trail" is named appropriately (Shoaf et al., 2004). Dudley Blane and his crew proposed a math route across Melbourne. Once schools started using this grade in math, it gained popularity. Vancouver, Boston, Philadelphia, and San Francisco followed the pilot's success.

We may create a digital math trail using cutting-edge technology (Cahyono et al., 2020; Cahyono & Ludwig, 2019; Ludwig & Jesberg, 2015; Zender et al., 2020). In this study, we utilized VR to build new math trails. These panels are replacing conventional interactive displays due to their realistic 3D imagery (Xiong et al., 2021). VR users with the right eyewear and input devices may interact with computer-generated worlds. Virtual reality is a potential tool for effecting pedagogical modifications in the classroom (Wohlgenannt et al., 2020). This study's main question is: How STEM-based math trails may be conducted virtually?

Cahyono, A. N. (2023). Virtual Reality STEM Trails: Exploring Math Trails with STEM Education Approach in a Virtual World. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 85–90). WTM. https://doi.org/10.37626/GA9783959872522.0.10

METHODS

Exploratory research with junior high school teachers and students was done to find the solution. We developed a VR STEM Trails App and conduct some pilots of virtual reality STEM Trails activities. Researchers observed students as they worked in each pilot and interviewed them afterward to get their feedback. Worksheets were reviewed, and comments were offered. Student responses were collected in a survey after they had finished the activities.

RESULTS

In this study, a model of the learning environment was designed by constructing math trails using a STEM approach in a digital world that was equipped with virtual reality technology. The application was designed to work on mobile devices accompanied with Cardboard VR (Figure 1). This application offers a learning environment in a virtual town with some landmarks that are most well-known in Indonesia. The math trail is comprised of the total of the outcomes of the many mathematical activities that are tied to the various landmarks along the path. Utilization of aspects of science, technology, engineering, and mathematics are required throughout the process of developing tasks. By using this application in combination with Cardboard VR, users can work together on their virtual mathematical trails.

Figure 1: The app and devices used in this activity.

Teachers may explore the virtual town with the help of Cardboard VR by utilizing the app to do so. They can then use the app to develop math trails projects related to the locations they find while doing so. The planning procedure for the task considers scientific, technological, engineering, and mathematical fields. The teacher has also completed their part of the project by offering the user assistance in finding a solution to the problem that they are now facing. Other methods have also been provided as part of the work that has been produced, and they form an essential part of this effort. After the task has been defined, the association uses group discussions to assess it. These discussions take place inside the association. Every teacher comes up with a task on their own, and then all those separate tasks, together with the ones that were produced by the other teachers, are put together to make a trail. There are numerous tasks that are developed; nevertheless, according to the results of the discussion, the ideal number of tasks that should be included in a trail is between four and five. This is the case even though there are many tasks that are generated. Figure 2 shows the activities conducted by the teachers.

Figure 2: Teachers' activities.

Student collaborates on projects with other classmates in groups of two to three students. Every group receives its own set of apps, Cardboard VR, and student worksheets. They collaborate with one another to solve mathematical problems while wearing Cardboard VR headsets and exploring virtual town. Students follow location markers that are posted at the curb at each junction or use accessible routes to help them trace math trails and find landmarks that are connected to topics they are working on in math class. This helps students discover landmarks that are connected to topics they are working on in math class. After doing a search for a location, it normally takes around a minute to find what you're looking for. After the students have determined the correct location, the following step calls for them to read the job description that is written on the board that is situated all around the landmark. Figure 3 shows the activities conducted by the students.

Figure 3: Students' activities.

When the user has discovered the position of the landmark that it is meant to send them to, a board will emerge that offers information on the action that has to be carried out at that spot. Users first collect data that has been available at that location to complete the activity, and then they solve the task that has been posed to them by following a mathematical modeling cycle. After leaving Cardboard VR, the user will start using the worksheets that have been provided as a supplement to this app.

Utilizing the worksheets, the user's objective is to not only record the data and information that was obtained but also the solutions to the problems that were encountered at the task site. They are going to make use of Cardboard VR again to proceed with their journey to the subsequent spot where they will complete a work if they have previously completed a task in the present location. These activities are shown in Figure 4.

As part of this implementation, students will work on an application for a length of time that is roughly equivalent to three minutes. They next put away the tool and conclude the project by having a conversation with their group and recording their work on a worksheet. After that, the tool is put away. They resumed employing the tool and continued their journey to examine the trail and discover answers to new challenges that they encountered along the way in other regions once they had successfully completed the mission.

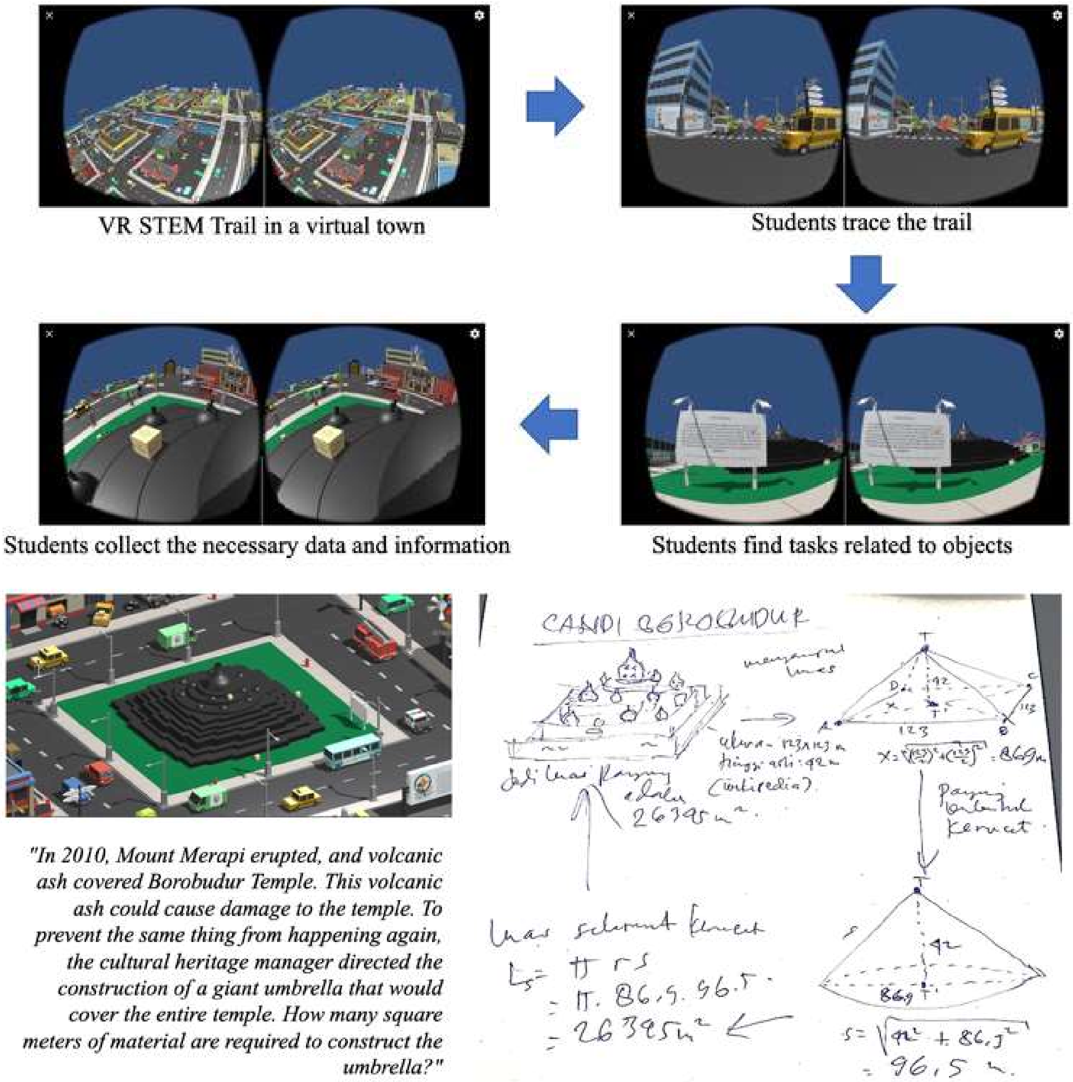

Figure 4: Learning activities.

In this activity, students learn about mathematics about surface area which is integrated with science related to the impact of volcanic ash from the eruption of Mount Merapi on buildings. This activity is also integrated with engineering by teaching students in designing building construction. The use of technology in this activity has been able to facilitate the integration of fields in STEM.

CONCLUSION

In this study, a VR app for mobile phones was developed that works with VR Cardboard headset. Plans for teaching and learning mathematics were also produced as part of this study. Teachers may create virtual reality environments with mathematical challenges based on landmarks in a made-up town. Like conventional math trails, students use a virtual reality app on their mobile devices to follow the path mapped out by the teacher. They run across STEM (science, technology, engineering, and mathematics) tasks along the trail, which they solve by using the mathematical modeling cycle. In a pilot program, students' mathematical modeling abilities for addressing real-world issues were enriched, and they participated more actively in mathematics-related activities. By using the advancements in technology, it can be done anytime and wherever. Large-scale trials are required, as are widespread implementations of the results.

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LEARNING BY TEACHING WITH VIDEOS - AN INTEGRATION STRATEGY TO PROMOTE MATHEMATICAL COMMUNICATION AMONG STUDENTS OF HIGHER EDUCATION

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Abstract. This paper intends to present the LIGHTS (LearnInG by teacHing wiTh videoS) project that aims to promote a better academic inclusion of new students in an engineering school (ISEP), in the sense of their full integration into the new academic life, making use of their mathematical knowledge and communication skills. The pandemic affected face-to-face communication and in-person socialization. We present the students' opinions about the challenge they faced when participating in the LIGHTS project during the Covid-19 pandemic.

Key words: Integration in higher education, LIGHTS project, mathematical communication.

INTRODUCTION

Participating in group work, helping colleagues to complete tasks, and interacting with the teacher are essential to facilitate teaching and learning. This interconnection is achieved in conventional classroom teaching, where students and teachers interact face-to-face, but it is minimal when students are involved in an online learning situation. The Covid-19 pandemic and the related measures and restrictions brought barriers to communication. The communication and interactions between students and teachers and between students and their peers have been directly affected (Aboagye et al., 2020). The challenge was enormous and, from one moment to the next, the educational community found itself in the middle of a tornado. The imposed restrictions to daily life have forced the educational community to quickly adopt different ways of working, learning, and connecting with each other. The pandemic affected face-to-face communication and in-person socialization. Students were jolted from their academic routines, and socialization and communication were affected.

The situation becomes more complex when it comes to 1st year students who are joining higher education. It is very important to promote the integration and socialization of new students coming to higher education (Rezaei, 2018; Vagos & Carvalhais, 2022). Many of these young students are displaced, that is, they are far from home, far from their comfort zone. When a young adult transitions to higher education, he/she starts a new phase in his life - it is a new period full of challenges, both personal and academic. That's why it's important to promote the integration of new students, help students interact with each other and with the teacher, even if the face-to-face contact time is very short. While this crisis has brought unprecedented challenges for both teachers and students, it has brought a wave of inspiring new ideas. New ways had to be found to communicate.

Caldeira, A., Lopes, S., Costa, A. R., & Figueiredo, I. (2023). Learning by Teaching with Videos - An Integration Strategy to Promote Mathematical Communication among Students of Higher Education. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 91–98). WTM. https://doi.org/10.37626/GA9783959872522.0.11

This paper describes a project that aims to:

- promote the integration of new students, help students interact with each other and with the teacher;
- improve mathematical communication skills;
- cultivate means that lead to academic success.

The paper is structured as follows. The next section contains background material: integration in higher education, mathematical communication skills and low-cost videos as a learning tool. Then, is described the LIGHTS (LearnInG by teacHing wiTh videoS). The results are presented later and finally the conclusions and considerations.

BACKGROUND

In this work there are three fundamental points to consider: integration in higher education; mathematical communication skills and low-cost videos.

Integration in Higher Education

When a young adult transitions to higher education, a new phase in his life begins; it is a new period filled with both personal and academic challenges. In adapting to higher education there is an academic aspect (study, curricular involvement, school performance, etc.) and a social aspect (relationship with teachers, colleagues and other elements of the educational establishment, participation in social activities, etc.), which is why a good integration can positively influence behavior and adaptation to the new context (Caldeira et al., 2016; Almeida et al., 2018). Academic integration can be defined as academic progress, cognitive growth and positive learning experiences (Tinto, 2015). Tinto argued that both personal and institutional strategies are equally responsible for adaptation in academic pursuit among students. In particular, personal strategies include skills, abilities, and previous education, as well as students' goals. Good integration into higher education helps students have better academic lives and not drop out of their studies.

Mathematical communication skills

Mathematical communication skills refer to the students' ability to (Rohid et al, 2019):

- arrange and link their mathematical thinking through communication;
- communicate their logical and clear mathematical thinking to their colleagues, teachers, and others;
- analyze and assess mathematical thinking and strategies used by others;
- use mathematical language to express mathematical ideas correctly.

Writing skills are an important part of communication. Good writing skill allow students to communicate their message with clarity and ease. Throughout our academic life, we are led to think that writing is more important than speech, as stated by Storto (2020). Maulyda et al. (2020) conducted a study focusing on students' mathematical communication skills whose objective was to describe the students' written and oral mathematical communication skills in solving word problem. They concluded that students' verbal communication skills are better than their written communication skills. As Ribosa and Duran (2022) state, science teaching and learning practices need to enable students with skills to talk about science, so that they can verbally formulate, exchange explanations to communicate and argue scientific ideas.

It is necessary to implement a more frequent and regular oral pedagogy in mathematics classes, which enhances moments in which students express themselves and interact, in order to develop essential skills for adequate oral expression in the formal context of communication in mathematics. For the student, the more he/she talks, the more he/she will gain talent, dexterity and skill in the art of speaking mathematics. It is necessary to develop teaching strategies that provide students with moments of communication to develop their critical and reflective capacity. Furthermore, one of the most important tasks of higher education is to prepare students to work in a changing world and to motivate them to develop skills other than those specific to their fields of study. Abadzi states that:

> Technological achievements and the globalization of labour require complex skills for the workplace. Companies reportedly demand employees ready to "plug and play", who are also creative, communicative, and collaborative. Accordingly, international agencies often advise lower-income governments to de-emphasize "traditional" book learning and use innovative pedagogies to teach the needed skills explicitly (Abadzi, 2015).

Low-cost videos

Technology has changed the teaching and learning scenario. Media, and the use of video in particular, contributed to the change of teaching, learning, studying, communicating, working. The use of video as an educational tool is reinforced when three elements are considered (Brame, 2016):

(1) how to manage cognitive load of the video;

(2) how to maximize student engagement with the video;

(3) how to promote active learning from the video.

The use of low-cost videos in this work it was very important because it is a cheap tool and is relatively easy to handle. Doing a low-cost video, make the students improve their learning by making them more autonomous and contribute to make them more motivated and, consequently, contribute to improve the learning by teaching process. In addition, this task encourage discussion between the teams' elements (Caldeira et al., 2020).

LIGHTS PROJECT

It is widely accepted that the use of new technologies is a supportive tool to improve the effectiveness of learning (Targamadze & Petrauskiene, 2010). Among these tools, video has been used in recent years to support student learning (Barford & Weston, 1997; Bravo et al., 2011). The use of videos engages and helps students retain knowledge, motivates interest in the subject, and illustrates the relevance of many concepts. The LIGHTS project that aims to promote a better academic inclusion of new students in an engineering school (ISEP), in the sense of their full integration into the new academic life, making use of their mathematical knowledge and communication skills. The LIGHTS project involves the use of videos where students have an active role in the video rather than a passive, contemplative role - students create a video about how to solve a concrete math problem.

The scenario

In September 2020, the School of Engineering of the Polytechnic of Porto (ISEP), started its academic year, using the following model:

- Theoretical classes: synchronous and online.
- Theoretical-practical classes: synchronous, and a combination of in-person and online lectures. Face-to-face classes were interspersed with online and distance classes: half of the class had face-to-face classes and the other half were at home and online.

The question is: *How to encourage formal oral expression, involving mathematical communication, when students are in an online learning situation?*

159 first year students participated in the LIGHTS project, of which 99 are students of the 1st year of the Degree in Electrical Engineering – Power Systems in the subject of Linear Algebra and Analytical Geometry and 60 are students of the Degree in Telecommunications and Informatics Engineering in the subject of Algebra (from now on, Linear Algebra and Analytical Geometry and Algebra are called Linear Algebra).

Project execution

It started by forming teams with two or three students and each team received its task, in this case related to Linear Algebra. For communication with the students, it was used the Moodle e-Learning platform providing supporting material and for the delivery of the final work. A proper space was created in this platform, accessible to all persons involved, to help the management of the process, the delivery of the final work and its logistics. In this experience the following six steps were performed:

Step 1: Form the team;
Step 2: Get the project worksheet (different for each team);
Step 3: The problem (analysis, discussion and resolution);
Step 4: Plans discussed with the teacher with particular focus on the scientific contents;
Step 5: Make the video;
Step 6: Delivery the final work in the Moodle platform.

Part of the Moodle page can be seen in Figure 1, where students can consult the videos. All videos are available, and all students can consult the videos produced by themselves and by their colleagues.

Students study to teach their colleagues, they present in the video the solution to the proposed problem so that their colleagues understand the resolution of the problem and the contents involved - students learn while making the video and while watching each other's videos (Figure 2).

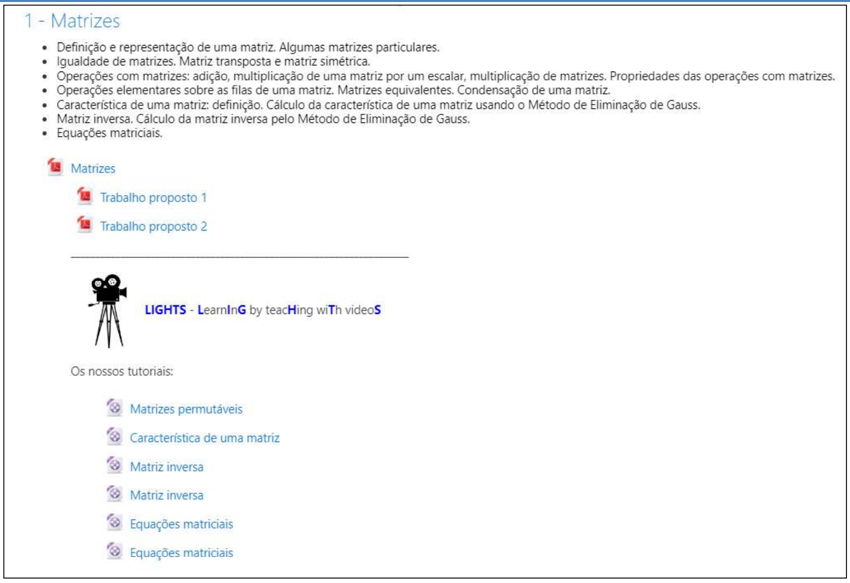

Figure 1: Aspect of Moodle platform (in this case about matrices).

The objective of this video is very specific and has a short resolution of mathematical problems. Students have to create it in a very short period of time (more or less 2 weeks) and with few resources. It is a low-cost educational video (Simo et all, 2010).

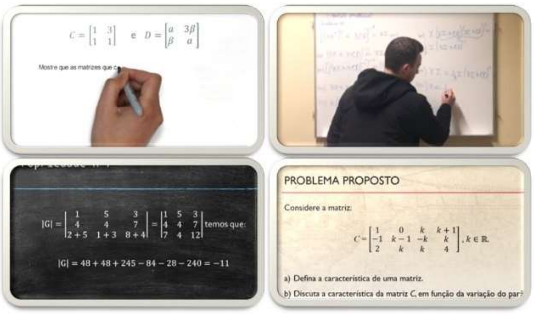

Figure 2: Visual aspects of some videos.

At the end of the project, the students' opinions about the challenge they faced were evaluated. A questionnaire where students were asked to give their opinion:

Q1: Despite the constraints due to the pandemic, I was able to integrate into the project and interact with my colleagues;

Q2: I think that learning/teaching through videos is a positive learning process.;

Q3: I was able to get involved in the project and promote my learning.;

Q4: I felt motivated and accompanied by the teachers during the course of the project;

Q5: I felt more motivated and interested in studying the syllabus;

Q6: I felt a greater development of knowledge and skills with the participation in the LIGHTS project;

Q7: I learned by watching my colleagues' videos;

Q8: The LIGHTS project should be applied in other curricular units.

RESULTS

These questions were evaluated using a Likert scale, with 1 being "I completely disagree" and 5 being "I completely agree" (see Figure 3).

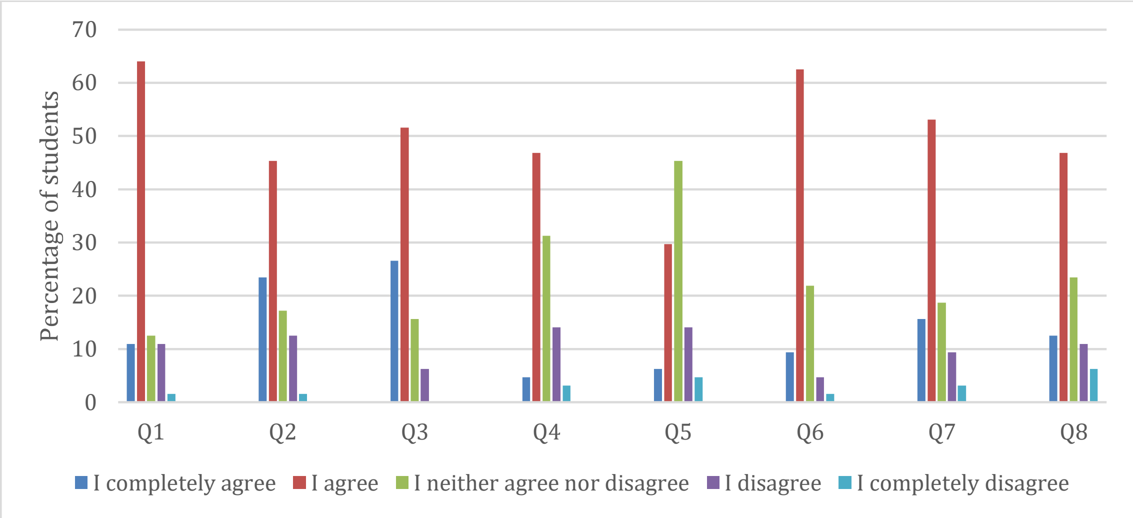

Figure 3: LIGHTS project students 'perceptions.

Sixty-four students answered the questionnaire. The average score was 3.6, considering all the questions, reaching 3.9 in the questions in which the students evaluated the project's objectives and 3.7 in the overall opinion of the project (see Figure 4). We highlight the average of 3.7 obtained when the students were asked about the importance of the project in their integration and in their interaction with their colleagues during the Covid-19 pandemic.

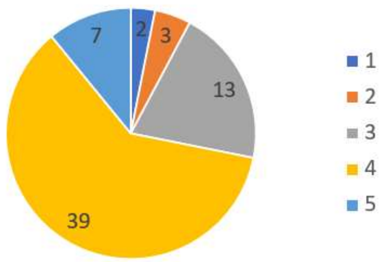

Figure 4: Overall evaluation of the LIGHTS project.

Students were also asked to provide general feedback on the project, as well as identify strengths and weaknesses. As for the strengths, the students highlighted the relevance of the project to learn and consolidate the concepts of Linear Algebra and the innovative way of learning in a team and training the ability to teach. Although most of the students did not mention any weaknesses, one team pointed to the fact that not all members of the group were equally committed.

CONCLUSIONS

The COVID-19 pandemic has created the largest disruption of education systems in history, affecting nearly 1.6 billion learners in more than 190 countries and all continents. Closures of schools and other learning spaces have impacted 94 per cent of the world's student population, up to 99 per cent in low and lower-middle income countries. (United Nations, 2020).

The COVID-19 pandemic has suddenly and abruptly forced schools and education to adapt to a new reality. COVID-19 has thrust digital technology and education into the spotlight. The integration of digital technology into the teaching and learning process has been fundamental, resulting in fundamental changes in education. The use of digital technologies gives teachers the opportunity to design engaging learning opportunities in the courses they teach.

A new and innovative way to integrate 1st year students of degrees of engineering in a pandemic time, improving their mathematical communication, is the LIGHTS project. The students developed videos using the concept of "Low-cost educational video" - they made a video with a specific goal and it was created in a very short period of time, with a few resource. In this way a number of common problems were eliminate: the budget and time decrease. With the LIGHTS project, communication paths were created between students and teachers and between all students.

The results obtained by the participating students were very positive. The students' involvement in the project was notorious. With this type of challenge, students had the opportunity to develop reflective thinking to overcome difficulties and thus develop knowledge and skills. In fact, many skills were developed by the students during the execution of this project. As an example, we can point to the ability to work with others, solve problems and conflicts and also the ability to make public presentations. This project demonstrated that the use of videos has a positive effect on students' motivation in learning, in this case Linear Algebra.

We can conclude that the students recognized the importance of this project not only to develop personal and teamwork skills, but also to consolidate the knowledge of Linear Algebra. In addition, it made the students not give up on the subject, accompanying it throughout the semester. Highlighting the fact that, even in a pandemic, it is possible for students to communicate with each other and with the teacher, interaction between all is possible.

As a final note, this project will be replicated at the University of Minho in Guimarães, Portugal, in Mathematics subjects and Engineering Degrees.

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PROJECT LEARN+: AN OPPORTUNITY TO DEVELOP MATHEMATICS LEARNING

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In this communication, we present the LEARN+ project and the teacher training model using the MILAGE LEARN+ platform for the teaching and learning of mathematics, which have been carried out by the Associação de Professores de Matemática (APM) from Portugal and the Federación Española de Sociedades de Profesores de Matemáticas (FESPM) from Spain, partners institutions in the Erasmus+ LEARN+ project.

Key words: Gamification, mathematics learning, MILAGE LEARN+.

THE LEARN+ PROJECT

The project *Building communities of teachers producers to implement personalized learning of Mathematics supported by machine learning and block chain to assess competencies* is an Erasmus+ project, known as LEARN+, with reference number 2019-1-PT01-KA201-061246, coordinated by the University of Algarve and its aim is to create a European network of teachers who use technology in the teaching of mathematics, in particular, using a specific platform, named MILAGE LEARN+, that also emerges from a previous European proposal, which offers personalised itineraries, thus catering the diversity of students' abilities. This platform facilitates the individualisation of the learning pathway and provides feedback logs that help teachers personalise their curriculum by adapting it to the needs of the students.

The project involves partners from Portugal, Spain, Cyprus and Germany, which, at the time of the joint proposal application, were all facing a common situation of declining students' interest in learning mathematics as well as low academic achievement in mathematics. The consortium includes 10 organisations from the above countries, which complement each other concerning their experience. In addition to the University of Algarve as the coordinating institution, it involves mathematics teachers' associations from Portugal, Spain and Cyprus, as well as a German association of mathematics and science teachers. Schools from Portugal, Spain, Cyprus and Germany are also involved, covering primary and secondary school levels, as well as a German Pedagogical Centre.

For many students, mathematics is boring, abstract, uncreative, complex and very difficult to understand. However, mathematical thinking is a competence that prepares and helps young people to analyse logically and accurately situations in daily life. The LEARN+ project seeks to help students achieve success in learning mathematics through the use of technology and gamification. According to Passey et al. (2004), the integration of ICT in the teaching and learning of mathematics helps to increase students' motivation, enabling self-paced learning. Gamification involves the use of the technique and concept of gaming in diversified contexts outside games (Deterding et al., 2011) and, when applied to the mathematics teaching-learning context, can contribute to increase students' engagement. It can encourage students to study and reflect critically, according to Alves and Teixeira (2014), through interaction

Carvalho, R., & Lázaro, C. (2023). Project LEARN+: An Opportunity to Develop Mathematics Learning. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 99–106). WTM. https://doi.org/10.37626/GA9783959872522.0.12

and collaboration. These authors also point out that gamification can "explore the cognitive, social, cultural and motivational qualities of students" (p. 140).

To achieve these goals, the LEARN+ project promotes mathematics learning supported by the creation of tasks, videos and gamification, with a scheme that guides both student self-assessment and peer-assessment, stimulating autonomous and active learning. The mathematical tasks posed and solved are shared through the platform by the teacher, so that he/she can also supervises them.

THE MILAGE LEARN+ PLATFORM

The MILAGE LEARN+ platform, the main tool of the European LEARN+ project, allows the extending of the traditional learning space to virtual learning environments in order to keep students connected, thus facilitating the implementation of a blended mathematics teaching and learning model. Gamification is also incorporated, integrating tasks associated with three different levels of complexity: initial, intermediate and advanced.

Thus, the great potential of this platform is to contribute to the improvement of the teaching and learning of mathematics, including all students. In this way, student-centred learning contexts are provided. Low-achieving students, who may have difficulties in learning the content covered in class, will not only have the opportunity to access the tasks set as many times as they need, taking advantage of mobile devices, but also have access to complex problems and activities, which also aim to stimulate high achievers.

In the MILAGE LEARN+ platform, there are two components: the teacher through a specific software that is installed on the computer and allows the creation and submission of materials (tasks and videos). This component may be considered as a back office, free for teachers and schools, who wish to join the development of content in mathematics teaching and the student's APP for mobile devices, such as tablets or smartphones enabling students to access educational content in and outside the classroom.

Figure 1 shows a summary diagram of the platform, followed by an explanation of the two components of the platform. The platform is being developed progressively by the University of Algarve in order to advance the integration of machine learning and digital certification, thus facilitating personalised learning and providing teachers with information about their students' learning progress. In addition, it favours the adaptation of a work plan to the specific needs of each student, providing different itineraries according to the characteristics required. In this way, the development of 21st century skills, such as creativity, critical thinking, collaboration, communication and autonomy are promoted. As mentioned above, in order to stimulate and support the implementation of the various activities proposed, the APP interface incorporates gamification features, segmenting different levels of exercise difficulty to support students with difficulties and also motivate more advanced students in learning mathematics.

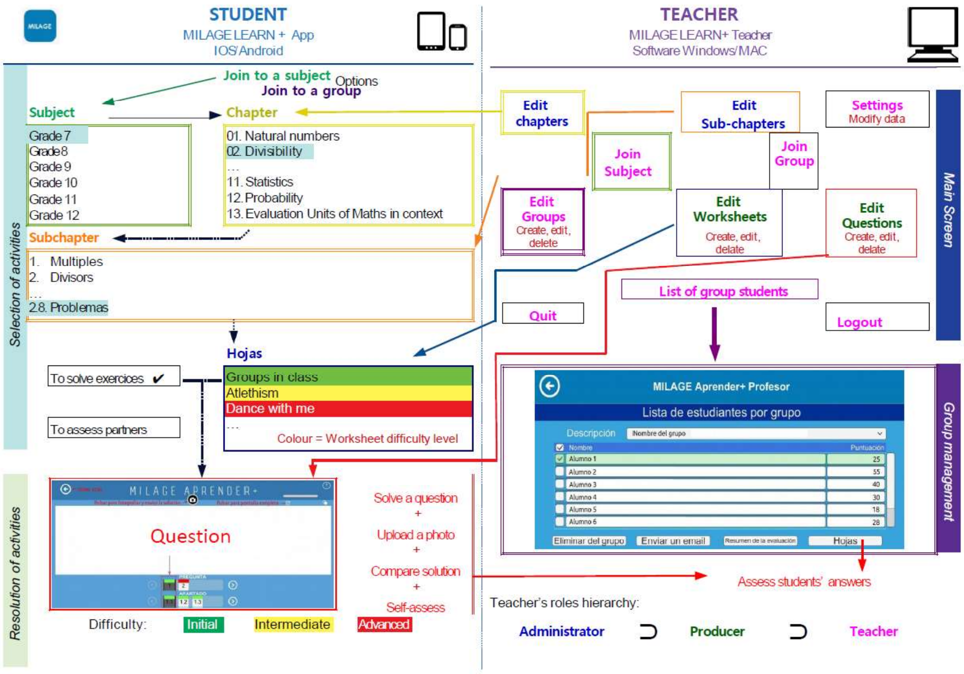


Figure 1: Overview of the MILAGE LEARN+ platform.

The MILAGE LEARN+ tool for the teacher creating materials

First of all, the teacher's software must be downloaded to the computer, with a Windows, Mac or Linux operating system, which can be accessed from the website http://milage.io by registering there. Once the software has been downloaded and installed, it opens and the following screen appears (Figure 2A) for registration. After registering and clicking on the access, there is a screen, which allows us to enter the activities on the platform (Figure 2B).

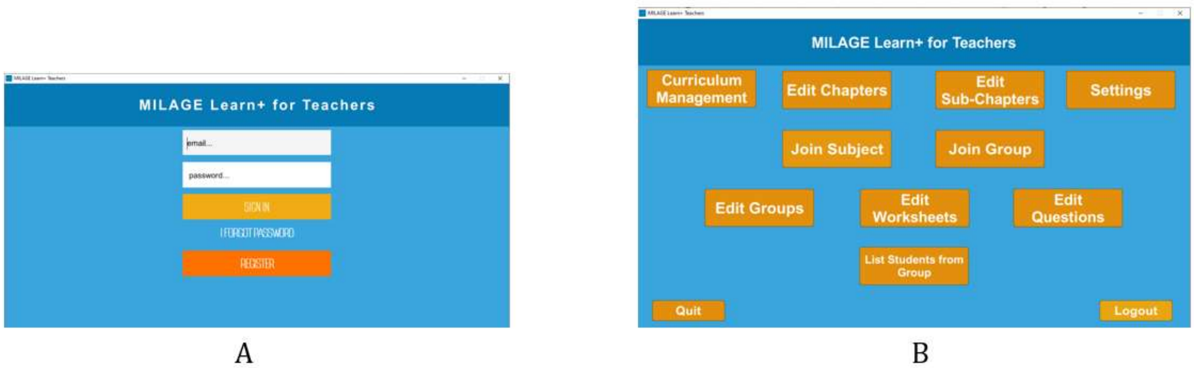
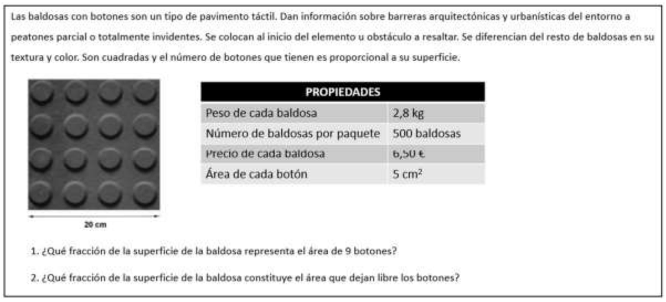


Figure 2: Main screen of the teacher's platform (A) and Screen to enter the activities on the platform (B).

On the screen (Figure 2B) where is possible to enter the tasks (worksheets) on the platform, the functions of the important tabs for the creation and submission of tasks are as follows:

- **Edit sheets:** gives access to the first level of task creation on the platform, i.e. the elaboration of mathematical tasks. First, select the level to which it is addressed (Subject), the subject to which it corresponds (Chapter) and the subsection of the chapter (Subchapter). Next, click on the circular plus icon, select the level of difficulty (Initial - green, Medium - yellow, Advanced - red) and type in a title for the worksheet.
- **Edit questions**: allow us to edit and associate questions to a worksheet already created. To create a question (problem) it is necessary to have an image file in PNG format with the questions, an explanatory video with the resolution of each of the questions and another image file for each of the solution and the scores assigned to each phase of the resolution process. Figure 3 shows an example of a problem with PNG files containing the questions and the solutions and scores for question 1 (Figure 4) and 2 (Figure 5).



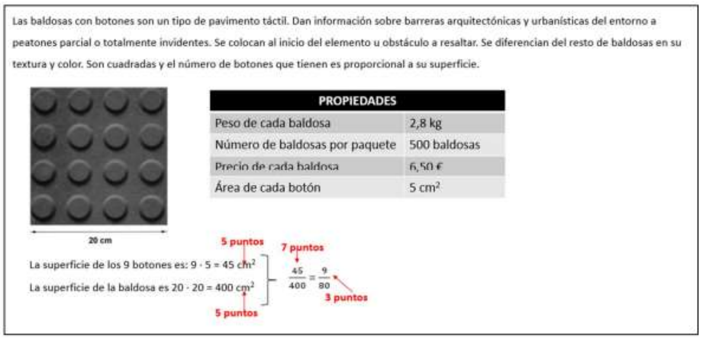
Las baldosas con botones son un tipo de pavimento táctil. Dan información sobre barreras arquitectónicas y urbanísticas del entorno a peatones parcial o totalmente invidentes. Se colocan al inicio del elemento u obstáculo a resaltar. Se diferencian del resto de baldosas en su textura y color. Son cuadradas y el número de botones que tienen es proporcional a su superficie.

PROPIEDADES	
Peso de cada baldosa	2,8 kg
Número de baldosas por paquete	500 baldosas
Precio de cada baldosa	6,50 €
Área de cada botón	5 cm²

20 cm

1. ¿Qué fracción de la superficie de la baldosa representa el área de 9 botones?
2. ¿Qué fracción de la superficie de la baldosa constituye el área que dejan libre los botones?

Figure 3: Example of an image of a question to be submitted on the platform.



Las baldosas con botones son un tipo de pavimento táctil. Dan información sobre barreras arquitectónicas y urbanísticas del entorno a peatones parcial o totalmente invidentes. Se colocan al inicio del elemento u obstáculo a resaltar. Se diferencian del resto de baldosas en su textura y color. Son cuadradas y el número de botones que tienen es proporcional a su superficie.

PROPIEDADES	
Peso de cada baldosa	2,8 kg
Número de baldosas por paquete	500 baldosas
Precio de cada baldosa	6,50 €
Área de cada botón	5 cm²

20 cm

5 puntos

La superficie de los 9 botones es: $9 \cdot 5 = 45\ cm^2$

La superficie de la baldosa es $20 \cdot 20 = 400\ cm^2$

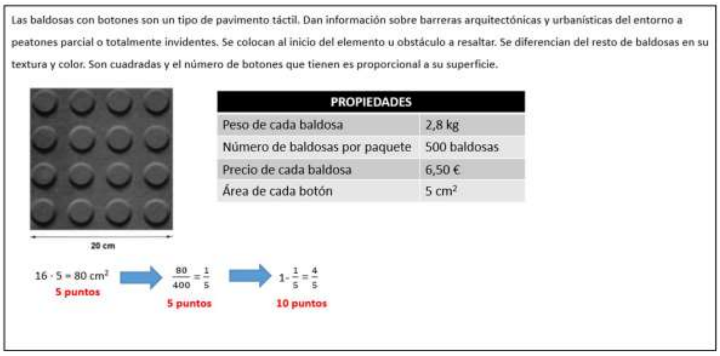
5 puntos

7 puntos

$\frac{45}{400} = \frac{9}{80}$

3 puntos

Figure 4: Example of an image of the solution and scoring of question 1.



Las baldosas con botones son un tipo de pavimento táctil. Dan información sobre barreras arquitectónicas y urbanísticas del entorno a peatones parcial o totalmente invidentes. Se colocan al inicio del elemento u obstáculo a resaltar. Se diferencian del resto de baldosas en su textura y color. Son cuadradas y el número de botones que tienen es proporcional a su superficie.

PROPIEDADES	
Peso de cada baldosa	2,8 kg
Número de baldosas por paquete	500 baldosas
Precio de cada baldosa	6,50 €
Área de cada botón	5 cm²

20 cm

$16 \cdot 5 = 80\ cm^2$ → $\frac{80}{400} = \frac{1}{5}$ → $1 - \frac{1}{5} = \frac{4}{5}$

5 puntos — 5 puntos — 10 puntos

Figure 5: Example of the solution image and scoring of question 2.

For the teacher to access all of the student's work, the student has to be enrolled in a group or class. The teacher only has access to what the student does within the group/class he/she has created. He/she has access to a variety of information about the student's work (e.g.

student scores or rankings) and can download a portfolio of all the student's work, or choose the time period in which the activities were completed. It is also possible to check and validate students' self-assessment and peer-assessment and change the score if necessary.

On the platform there is a chat through which the teacher can communicate individually with the students or with the whole group.

The MILAGE LEARN+ APP for students

The platform allows students to register (Figure 6) and access the learning content created through the teacher component.

The MILAGE LEARN+ APP, which is free and available for Android and Apple iOS systems, is a support tool for students, offering them the opportunity to independently solve worksheets with different levels of difficulty. It is also a support tool for teachers as it allows them to manage better the classroom time, in a sense where the teacher no longer has to provide the solutions to the tasks in class, as they are integrated in the application.

Once the student accesses a section of a proposed worksheet, he/she tries to solve it in his/her notebook and when he/she considers it is solved, he/she takes a photo from the mobile device, which is sent to the platform. Currently, the platform allows students to answer several types of questions such as true/false, multiple choice, short answer or text questions and to send a PDF file. After submitting their answer, the student can access the video explaining the resolution of each question and the image with the scores assigned to the different steps of the task solution process.

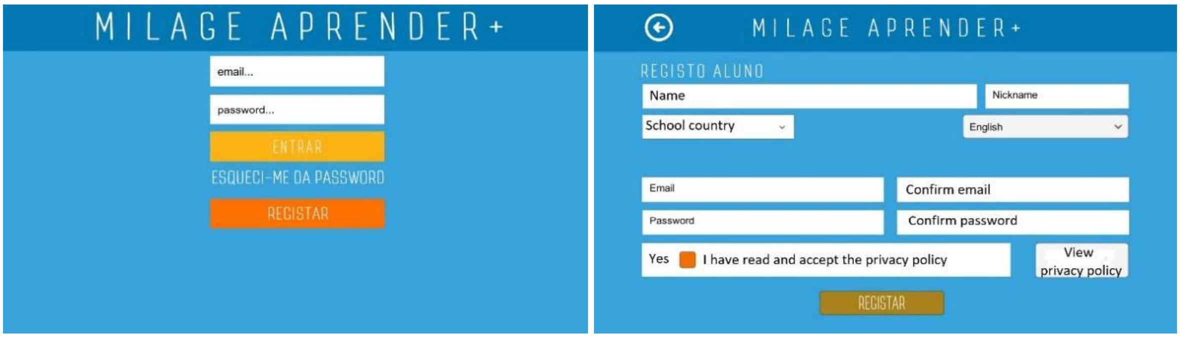


Figure 6: Main registration screen on the student platform.

In order to stimulate and support the implementation of the various activities proposed, the MILAGE LEARN+ APP interface incorporates gamification elements. Students receive points for solving the activities correctly and for assessing other students' solutions, an innovative aspect of the platform. In addition, the APP includes a self-assessment and a peer-assessment system to encourage students to work autonomously. After submitting their resolution in the APP, the student has to grade their answer and grade the answer of another student in their class, which is assigned randomly by the platform. Only after the self-assessment and peer-assessment, the student has the full score of their work.

TEACHER TRAINING

Professional development courses for teachers, as users and as content creators for the MILAGE LEARN+ platform were integrated into the Erasmus LEARN+ project. These training courses were planned and carried out differently in Portugal and Spain, as it will be explained in the following sections.

The case of Portugal

In Portugal, teacher training courses were carried out in two modalities: 4 and 6-hour workshops and 30-hour courses. In the workshop modality, teachers first complete a 4-hour training "Learning Mathematics with the MILAGE LEARN+ APP", where they learn how to use the platform as students (session 1 with 2 hours) and as teachers (session 2 with 2 hours) using mobile phones and computers. Only those who completed this 4-hour training session could attend the following 6-hour training "Teacher Authors: a strategy to promote mathematics learning with the MILAGE LEARN+ platform", which aims to create content for the MILAGE APP. All of the courses were done remotely, through the APM’s Moodle® and Zoom® platforms.

In the 6-hour workshop (three sessions with 2 hours each), teachers go through several phases: (1) they have to create a worksheet that has up to four questions and send it to the teacher educators with the resolution and proposed score to each question, to receive feedback on the didactic and scientific aspects of the worksheet content and format of the text; (2) after receiving the feedback, they make the necessary improvements and go on to create PNG images of the questions and resolutions and a video, which they send back to the teacher educators to receive feedback on the technical aspects to be improved in the images and videos, the didactic, scientific and linguistic aspects; (3) after receiving the feedback, they have to improve their work and make the remaining videos (one explanatory video for each question). Phase (1) takes place after session 1 of the training, where it is explained how to build a worksheet for the MILAGE APP. Phases (2) and (3) take place after training session 2, where it is explained how to make an explanatory video. In session 3, teachers must have all the material prepared and ready to be uploaded to the platform (PNG images of the questions, resolutions with scores and an explanatory video for each question). Only in this session, teachers have access to producer privileges and can, in the future, create and submit content into the MILAGE LEARN+ APP autonomously.

The 30-hour courses have the same the dynamics as the 4 and 6 hours workshops. However, in the courses, teachers have to do two worksheets and not just one, to review the work of other teachers (peer-review) who are in the training, and to use the APP with students in the classroom and share the experience. All the training courses were carried out by the Associação de Professores de Matemática (APM) in Portugal integrated in the LEARN+ project and were prepared and implemented with the aim of accompanying and supporting teachers in the use and creation of content for the MILAGE APP, so each workshop or course had a duration of about 3 months. During the LEARN+ project (2019/2022), APM has supported and certified more than 1100 teachers.

The case of Spain

In Spain, during the three years of the LEARN+ project, four courses of 30 hours each have been held. These courses have been organised by the FESPM and have been recognised by the Ministry of Education and Vocational Training. All of them have been done remotely, through the FESPM's Moodle® platform. From the second edition, the teacher educators offered streaming sessions with the participants, not so much to explain how the MILAGE LEARN+ platform works, but rather to disseminate the LEARN+ project and to share the experience of the partner school in Spain (i.e. IES Jesús de Monasterio), to encourage them to incorporate the MILAGE LEARN+ platform into its teaching practice, as well as to bring the support of the teacher educators closer to the participants. In practice, the main support was carried out through the answers to queries in the forums and the evaluation of course assignments with the required feedback.

Unlike in Portugal, in Spain, the MILAGE LEARN+ platform did not include at the beginning of the LEARN+ project tasks covering compulsory education contents, so the first two editions of the courses focused more on the APP part of the teacher as a producer of resources on the platform. In addition, the FESPM opted for a mathematics in context model for Compulsory Secondary Education. Therefore, the worksheets with tasks had to follow this schedule, which resembles, to a certain extent, the PISA model in the mathematical competence assessment units, presenting a stimulus and questions from different blocks of mathematical content. From the second edition of the course onwards, a topic was introduced in the course on classroom proposals with materials from the MILAGE LEARN+ platform, in order to promote its use. In the last edition of the course, it was considered necessary for teachers to begin by first getting to know the students' APP and then, the aspect of the platform for creating resources for teachers, with the following structure of topics covered in the training courses: 1) Carrying out activities on the MILAGE LEARN+ platform (student APP); 2) Tasks for the development of mathematical competence; 3) Resources for the creation of educational videos; 4) Creation of tasks to the MILAGE LEARN+ platform and 5) Classroom proposal with tasks from the MILAGE LEARN+ platform. Each topic was valued at 6 hours of duration, including the work on the activities to be presented in each one. In total, the FESPM training has been followed by 235 teachers in the four courses offered.

TEACHERS' FEEDBACK

The evaluation of the teachers who have taken the training courses is very positive, in general. Spanish teachers highlight the fact of getting to the platform and its possibilities, since they appreciate as a very positive aspect that it promotes student autonomy, as well as the possibility of developing attractive activities for students. In addition, they consider that the materials created by FESPM are of high quality, promoting the development of mathematical competence. Finally, they also consider the interactivity between mobile phones, videos and mathematics to be of interest.

In Portugal, teachers emphasize the usefulness of the platform. It is easy to use in face-to-face and online context; the teacher has access to students' resolutions in an easy way; it motivates students to learn Mathematics; and self and peer assessment give students the opportunity to look back to the task and reflect on what they did and think about different resolutions.

In both countries, some teachers feel that the development of videos takes them too much time and Spanish teachers would like to have an even greater number of activities on the platform.

In the LEARN+ project, several professional development courses for teachers were held in Portugal and Spain to present the functionalities of the MILAGE LEARN+ platform, to encourage teachers to develop materials and to use the APP with their students. Teachers in Portugal and Spain took up the training courses differently, and in Portugal the uptake was higher than in Spain. However, it should be noted that the MILAGE LEARN+ platform has been running in Portugal since 2016, and Spain only joined in 2019 and that, given the pandemic situation in 2020 and 2021, its use was supported by the Portuguese Ministry of Education, which led to more teachers participating in the training.

Acknowledgment

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TASKS AND LEARNING PATHS IN ASYMPTOTE AND GEOGEBRA

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Abstract. Automatic tutoring systems allow teachers to design tasks to assist students' learning processes. Here we analyse mathematics automatic feedback tasks built with GeoGebra and Asymptote; each task included a series of decisions about pedagogical and curricular strategies inherent to the tasks proposed, related whit a hypothetical learning trajectory. In this exploratory study, we aim to understand views about ATS by analysing data collected from six future teachers and two secondary students. The results reveal that the analysed tasks and learning graph were adjusted, provoked interest in the students, and there is evidence to have triggered a-didactic and mathematical situations with user engagement. Although automatic feedback GeoGebra tasks can provide several mathematics representations and ways of user interaction, they require more creator domain technology; hence, constructing the learning graph is easier with Asymptote.

Key words: Asymptote, automatic feedback, GeoGebra.

INTRODUCTION

Automatic tutoring systems (ATS), such as Asymptote, and other free software, such as GeoGebra, allow teachers to design tasks to assist their students' learning processes. In general, in these systems, the students can receive certified feedback on their answers and access hints that may allow them to overcome some difficulties. In these systems, the creator of the tasks intends to create a scenario with a teaching objective, idealising a learning path and parameterising the system to adjust it to his initial idea. The process of designing learning tasks in learning situations in face-to-face teaching is complex, but it relies mainly on the interaction between the teacher and students. Hence, the communication established in the classroom allows the adjustment of the task and adaptations to the students. In ATS, the interaction between students and the system cannot be monitored in the same way as in face-to-face teaching, which entails new challenges for the teacher in creating tasks with automatic feedback.

To reflect on the construction of tasks for teaching mathematics with automatic feedback, which can be inserted in ATS, we present the theoretical framework that guided this exploratory study that aims to understand future teachers' and secondary students' views about ATS, considering GeoGebra and Asymptote. Next, we present two tasks, one built in GeoGebra and another on the Asymptote platform, created to provide automatic feedback to users guided by the theoretical considerations presented. We also offer in this study the results of the first application of these tasks to two secondary students, as well as the perception of six future teachers about them. Finally, we discuss the results obtained and establish some final remarks.

Dos Santos Dos Santos, J. M., & Carvalho e Silva, J., & Lavicza, Z. (2023). Tasks and Learning Paths in Asymptote and GeoGebra. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 107–114). WTM. https://doi.org/10.37626/GA9783959872522.0.13

THEORETICAL FRAMEWORK

This paper only analyses the application of ATS for teaching and learning mathematics. We are creating a system that privileges the occurrence of mathematical situations, which places the student in a mathematical activity without the teacher's intervention (Brousseau, 2010, p. 21). The tasks constructed in these ATS are situated in didactic situations, thus,

"the teaching and teaching-learning processes to apply, require that the teacher provokes the student – through the sensible selection of the 'problems' they propose – adapting them to teacher intention. Such problems are chosen so the student can accept, make, using his own dynamics, that the student act, speak, reflect, and evolve" (Brousseau, 2010, pp. 34–35).

Along any didactic situation occurs an a-didactic situation that corresponds to certain moments of the learning process in which the student was not suffering any kind of direct control of the teacher about the mathematical content (Freitas, 2008, p. 84). In this way, a mathematical learning task in an ATS corresponds, in the first phase, to the didactic situation; in the second phase, the didactic situation made will trigger a set of a-didactic situations, ultimately perhaps in greater numbers than in face-to-face teaching.

Another relevant issue in the tasks designed in an ATS is to understand the extent to which they allow the self-regulation of student learning, which according to Leonor Santos and Paulo Dias (2013), "is the ability of the student to evaluate the execution of a task and make corrections, when necessary". In ATS, within each task, the feedback that can be given to each user's interactions corresponds to a strategy of self-regulation of student learning. However, the effectiveness of feedback depends on its opportunity and relevance. Regarding the principles of good feedback practices, Nicol & Macfarlane-Dick (2006) identify seven principles of good practice: i) help clarify good performance (goals, criteria, expected standards); ii) facilitate the development of self-assessment (reflection) in learning; (iii) provide students with high-quality information about their learning; vi) encourage dialogue between teachers and peers around learning; v) encourage positive motivational beliefs and self-esteem; vi) offer opportunities to reduce the distance between current and desired performance; vii) provide information to teachers that can be used to help shape teaching (feedback). These principles seem to us to be adopted when we think of a type of feedback that goes beyond the feedback of the mere certification of students' responses.

There are studies that have focused on the use of automatic feedback associated with presenting user scores in the correct responses and resolution times and in the sense that students can conduct self-assessment processes, particularly in problem-solving (Drijvers, 2018; Barana, et al., 2022). Recent studies have used GeoGebra's automatic demonstration capabilities to promote activities with automatic feedback on geometric and algebraic geometry problems (Kovács et al., 2020; Kovács et al., 2022). Issues related to the creation of automatic feedback in teacher training have also been addressed (Hašek, 2022). In the case of the project "GeoGebra as a strategy for remote teaching: creating activities with automatic feedback" (Dos Santos et al., 2022), the design of the tasks intended to provide feedback from a detailed analysis of a set of previously listed plausible student resolution strategies, with the teacher being the key to creating the appropriate feedback for the user task progress and, in a way, overcome the difficulties eventually reviewing or seizing concepts necessary for the challenge posed.

MATHEMATICS AUTOMATIC FEEDBACK TASKS

Tasks with automatic feedback can be defined as micro tutorial systems and could be integrated into an ATS. In general, the design of these tasks is quite sensitive, mainly if traditional techniques are used for systems based on rules created by experts (Inventado et al., 2017). The process of learning tasks design in face-to-face teaching is complex, but it relies on the interaction between the teacher and students. Hence, the communication established in the classroom allows the adjustment of the task and adaptation to students. Generally, in ATS, the interaction between students and the system cannot be monitored in the same way as in face-to-face teaching, which entails new challenges for the teacher or creator in these tasks. Let’s review how this can be done, with some advantages, in the ATS we have been discussing.

Automatic Feedback Tasks in GeoGebra

Considering the automatic feedback tasks in GeoGebra, the feedback must be designed by an expert who is familiar with the mathematical contents, both from the didactic side as with the technological side. In particular, the technological domain of GeoGebra is fundamental to the design of these tasks. In addition to the correct answer, feedback could be provided by text images or sounds and incorporate other sub-tasks that allow the user to overcome the difficulties encountered in the face of an incorrect answer. One of the characteristics of tasks created with automatic feedback in GeoGebra is that the parameters can be randomly changed, so each task has an associated number of utterances that can vary in each user access.

An example of a task with automatic feedback, built with GeoGebra, of problems with words for the addition of natural numbers in the 1st year of schooling is shown in Figure 1.

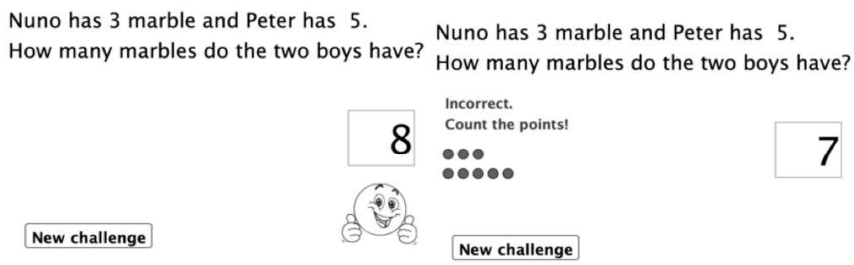

Figure 1: Word problems and adding integers in (Dos Santos et al., 2022, p. 79).

The option of the teacher was to give feedback with a static image in case of a right or wrong answer. In the case of the wrong answer, the feedback strategy is also visual, but in this case, it is dynamic because it depends on the utterance that changes each time the user presses the new challenge button. It should be noted that this feedback is based on the didactics inherent in adding natural numbers in the first year of schooling, first supported by the image students can use subitizing, and if they are unable to do so, they can count the points (Benoit et al., 2004), although this is a strategy to be gradually abandoned.

For each task, the creator, as an expert in content and didactics, builds an analysis matrix of the task, where he anticipates the possible answers of the students and elaborates feedback according to their didactic knowledge and the response given by the user (see the left side of Figure 2). At a later stage, a process diagram is drawn up that conducts the application's design in GeoGebra, in addition to schematising the actions to be taken in the face of the user's response (see the right side of Figure 2).

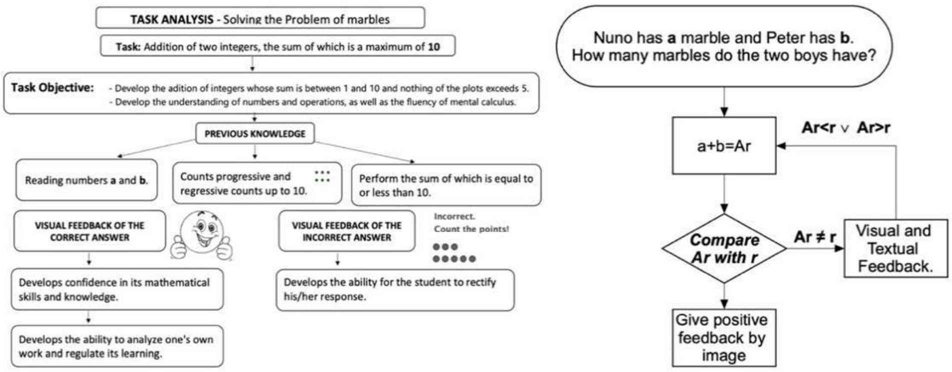

Figure 2: Matrix and Process Diagram of the task Word problems and adding integers in (Dos Santos et al., 2022, p. 80).

Although this example does not have subtasks, in the project "GeoGebra as a strategy for remote teaching: creating activities with automatic feedback", other activities were created where subtasks were used. Different types of inputs were used; the user, in addition to filling in text boxes, could fill tables; represent points and lines graphically. These actions are verified by the application and given the respective feedback.

Automatic Feedback Tasks in Asymptote

In the case of the Asymptote platform, in the 2022 version, the tasks have associated: a static image, a question whose answer may be multiple choice, a range, or a short answer. The task response or a set of clues, limited to three, can be given by text or a static image. Within each task, the user may have a maximum number of successes related to the number of clues given. By associating a set of tasks listed by a learning path previously designed by the task builder, we can establish a hierarchical order between tasks; some are called support tasks, and others are improvement tasks. The success or failure of the user in the learning path is that it determines the need to access the support tasks or be able to move forward to the challenge tasks. An example of a task with automatic feedback built-in Asymptote is Figure 3, which presents the first task built by one of the authors of this paper.

During the application design process, a first stalemate arose; the author of this task intended the image to be in GIF format, thus highlighting the dynamism of the task, whose response corresponded to a range of real numbers. However, the image could only be static. Other details were followed that have been overcome about the use of Latex and the natural learning process that the Asymptote platform requires. Later, in the process of submission of the task, it was necessary to change its categorisation to reasoning task, at the suggestion of the review process, a situation that led the author to change his strategy about the activities to begin to build in the portal for the construction of the learning path, schematized in Figure 4.

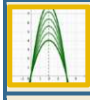	Families of Parabolas Main tasks Task category: Reasoning	
Task	The image depicts a family of parabolas passing through coordinate points (0.0) and (4.0). Consider point A, which belongs to one of the elements of this family, with abscissa 2 varying its order in the interval [4,8]. Knowing that the equations of this family can be described by equation y=α(x-0)(x-4), with α∈R, what is the range of variation of the α?	
Answer	**[-2 , -1]**	
Sample solution	Replacing x by 2 in the parabola family equation, we will have y=-4α. The vertex of the parabola family corresponds to point A, by which 4<-4α<8. Dividing inequalities by -4, we get that -2<α<-1.That is α∈[-2,-1]	
Stepped Hints	1	Note that point A corresponds to the vertex of the family parables
	2	Calculate the generic value of the abscess of A using the parabola equation. It is impossible to see that the expression of the abscissa of A framed between 4 and 8 allows obtaining the values of α.
	3	You have to solve two inequations, 4<-4α and -4α <8. It observes that multiplying or dividing both members of an inequality by a negative number changes the signs of inequalities.
Explanation	Replacing x by 2 in the parabola family equation, we will have y=-4α. The vertex of the parabola family corresponds to point A, by which 4<-4α<8. Dividing inequalities by -4, we get that -2<α<-1. That is α∈[-2,-1].	

Figure 3: Summary of the task "Parabolas family" available at https://www.asymptote-project.eu/en/portal-en/#!/task/t181992.

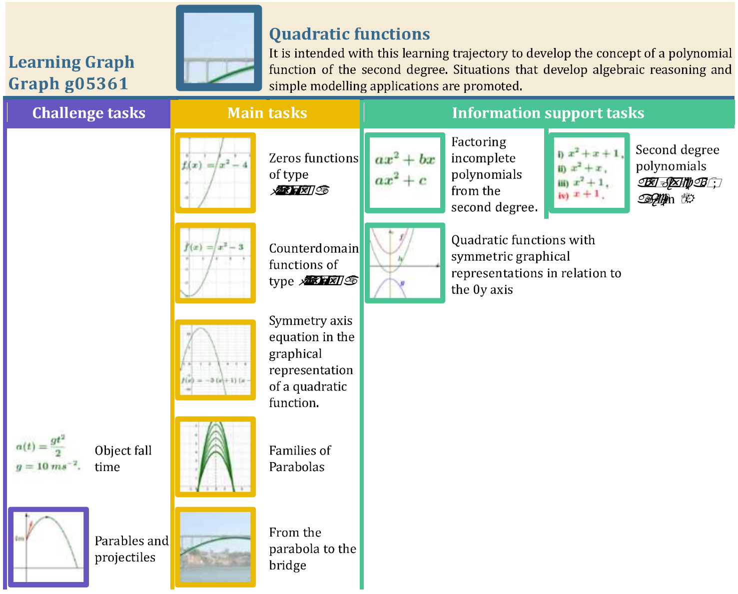

Figure 4: Tasks in learning graph "Quadratic functions" available at https://www.asymptote-project.eu/en/portal-en/#!/graph/g05361.

The learning path that came to be defined by the author involved in the construction of the previous task began to include five main tasks, three informational support tasks, and two challenge tasks.

In the learning graph, tasks were designed to have several ways to answer as fill-in, multi-choice, and value-indicating responses. Support tasks never included a specific user action other than reading the question, which included information and a series of questions for the user to think about the answers without them being directly asked for answers. The author understood that support tasks did not require specific user actions. Later the author came to know that this was different from the understanding of many other users of the platform because, in general, they put the student in need to interact with the platform, answering the support task.

VIEWS OF FUTURE TEACHERS AND STUDENTS

Tasks with automatic feedback in GeoGebra have been implemented in several classes of the 1st cycle of basic education in Portugal, these are prepared by the teachers, with interesting results, namely the one presented in Figure 2. These kinds of applications promote the autonomous work of students, with improvements in performance, being the technological context motivating and facilitating students, who interpret the work as a game (Alves et al., 2022). The teachers involved in the construction of these tasks considered it necessary to improve their technological knowledge, but at the same time, they mentioned that their investment was very positive in view of the results obtained.

Regarding the task and Learning Graph of the Asymptote platform presented here, it has not yet been tested with many students. In the review process, all tasks were accessed and made public. To perform a validation of the tasks and the learning graph, it was requested that two students in the last year of secondary school, Group I, and six students in a master's course on the teaching of mathematics in the third cycle of primary and secondary education, Group II, collaborate. Two virtual classes were created, in which the students performed the Learning Graph tasks and were asked to present their comments and opinions later.

Related to Group I, the two secondary school students considered that: “the Asymptote app is interesting and intuitive; when we work on the tasks seemed to be like playing a game; so, it became more fun to solve exercises”. One of the students also points out

> "It is very good to be able to perform exercises with the essential challenges to learn the selected subject, but at the same time to be able to learn a little more about subjects that would be given later with the extra challenges."

There was only one suggestion for improvement in mathematical writing "in our mobile phone the square roots did not cover the numbers". It should be noted that during the tasks the students used paper and pencils to develop strategies and calculations and in addition, they established mathematical dialogue about the tasks.

The six students in Group II, future mathematics teachers, considered the tasks interesting and challenging. Three of them mentioned that they would consider using the Asymptote platform in practical classes with their future students. All of them pointed out that the platform seemed interesting to use, because it allows the creation of a kind of quiz, illustrates the functions, and not imposing a restricted time for students to perform the tasks. Regarding the learning graph tasks, all mentioned that the clues were useful in the quadratic part and in recalling certain properties of the parabolas. One of the students also mentioned: “I found the tasks interesting and challenging, covering several areas: quadratic functions (analysis) and projectile launches (physics)”. Like what happened in Group I, during the tasks, group II students also used paper and pencils to develop strategies and calculations: in this regard, one of the students states: "I felt that the tasks throughout the questionnaire were slightly increasing the degree of difficulty. It will be easier through a draft to come up with solutions by doing our auxiliary sketches and calculations."

Although less frequent than in the case of the dyad of Group I, some discussions about the tasks were also established in the dyads of Group II.

DISCUSION AND FINAL REMARKS

On both platforms, feedback is provided to the user; however, the opportunity and quality of the feedback are always related to the hypothetical learning trajectory (Simon, 1995) built by the teacher, essential for creating an automatic tutorial microsystem, which makes the student's learning path more effective. Thus, the feedback provided in the task, the hints or option to provide other additional tasks, is always guided by a hypothetical learning trajectory, where the teacher or programmer of the tutorial microsystem equates the thought and learning of students in the activity with feedback that creates. In the case of the face-to-face relationship in the classroom, Lurdes Serrazina and Isolina Oliveira (2010) that hypothetical learning trajectory guides teaching for the understanding of students or users of the application. Also, in the case of the Asymptote platform and GeoGebra's tasks with automatic feedback, the hypothetical learning trajectory previously defined is fundamental for creating tasks to get opportune and good feedback. Also, in the learning graph built on the Asymptote platform, presented here, the hypothetical learning trajectory was essential to achieve the objectives proposed a priori.

In the case of the automatic feedback tasks built with GeoGebra in the projects coordinated by Abar et al. (2022), the automatic feedback tasks were created according to the seven principles enunciated by Nicol and Macfarlane-Dick (2006), Each task included “a series of decisions by the author about pedagogical and curricular strategies inherent to the proposal, and also about the programming strategies in GeoGebra that they would use” (Abar et al., 2023, p. 24).

Although the number of participants is very small, the results reveal that the learning graph was adjusted, provoked interest in the students, and there is evidence to have triggered a-didactic situations in a sense defined by Brousseau (2010). It should be noted that Group II participants already knew and had studied the mathematical contents of the learning graph, but their records and the discussions they established allow us to affirm that students were involved in mathematical situations.

When we analyse the task production methods and learning paths in both platforms, we learn that the environments are engaging for users, largely by the fast way they have feedback, which in somehow makes the tasks look like a game. Clearly, in both cases, the key is who creates the content instead of a teacher. The process of creating content on both platforms will depend on the contribution of users, posing enormous challenges in the evaluation processes of these materials.

Considering the preparation of the Learning graph is simplest in the Asymptote platform since the user only must relate to previously elaborated tasks. However, tasks with automatic feedback in GeoGebra can be more user-effective, providing a wider number of mathematics representations and different ways of user interaction, but requiring much more domain technology by the content creator.

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DIGITAL SUPPORT OF MATHEMATICAL MODELLING: THE ROLE OF HINTS AND FEEDBACK IN MATHCITYMAP

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Abstract. Mathematical modelling addresses numerous demands of modern mathematics education: The learning of mathematics is directly linked to reality and students learn impressively what they can use mathematics for. At the same time, these tasks are very challenging for students, especially when the tasks need to be structured and validated without the immediate help of a teacher. Modern approaches, such as MathCityMap, pursue the support of modelling activities with digital technologies. In this paper, the potential of MathCityMap features like hints, solution validation, and sample solution are explored in modelling tasks in different contexts. The results show that especially the solution validation feature makes clear that sometimes a second run of the modeling circle may be necessary. Furthermore, practical observations on hint usage emerge, which should be considered in future activities, i.e. in terms of modelling activities in the context of distance and online education.

Key words: Digital tools, MathCityMap, mathematical modelling.

MATHEMATICAL MODELLING WITH DIGITAL TOOLS

As one of the general mathematical competencies, mathematical modelling has found its place in the mathematics curriculum. Precisely because of its indispensable relation to reality, modelling distinguishes itself from the classical inner-mathematical problem and is thus intended to clarify the application relevance of mathematics (Blum & Leiß, 2007). Idealized, the modelling process is described as a cycle. The modelling problem usually derives from reality, i.e. a real situation. In order to work on this problem mathematically, students have to (1) understand the problem and (2) simplify it by choosing important information. By mathematizing the problem (3), students transfer it into the world of mathematics to afterwards work on it mathematically (4). The gained result is transferred back to reality by interpreting (5) where it is validated (6) and presented (7) thereafter (Blum & Leiß, 2007; cf. Figure 1).

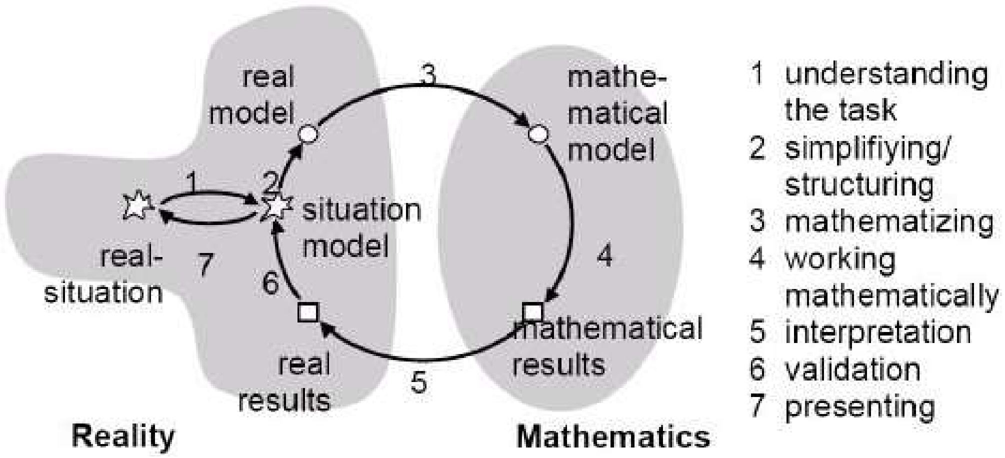

Figure 1: Modelling Cycle according to Blum & Leiß (2007).

Jablonski, S. (2023). Digital Support of Mathematical Modelling: The Role of Hints and Feedback in MathCityMap. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 115–122). WTM. https://doi.org/10.37626/GA9783959872522.0.14

From empirical studies, it is known that mathematical modelling presents students with particular challenges in the following steps of the modelling process (Schukajlow, 2010):

- reading and understanding the task (1),
- recognizing the connection between facts of the situation and the mathematical solution structure (2 and 3),
- transforming mathematical structures (3), performing the arithmetic operations (4), and interpreting the results (5).

Additionally, empirical studies show that independent validation processes (6) rarely take place in students' modelling activities (Hankeln, 2020).

The challenges here described can lead to students being overwhelmed in solving the modelling problem satisfactorily (Niss & Blum, 2020). A support possibility of students in mathematical modelling is realized through the use of digital tools. Greefrath and Siller (2017) describe digital tools as a potential aid for teachers and learners, especially in the context of modelling problems. Depending on the tool and its purpose, different steps of mathematical modelling can be supported by digital tools, e.g. validation. By providing feedback on a given answer, the use of digital tools could promote and support these important mathematical activities. In general, there are two possible advantages of using digital tools in the modelling process: digital tools can be directly implemented into the steps of the modelling circle on the one hand, and on the other hand, they can elicit the execution of the individual steps in the modelling process.

FEATURES OF THE MATHCITYMAP SYSTEM

With MathCityMap, outdoor mathematical tasks can be created and solved in the context of a mathematics trail (Ludwig & Jesberg, 2015). While running a mathematics trail, students work in small groups and follow a route leading them to mathematics tasks being linked to real objects and situations.

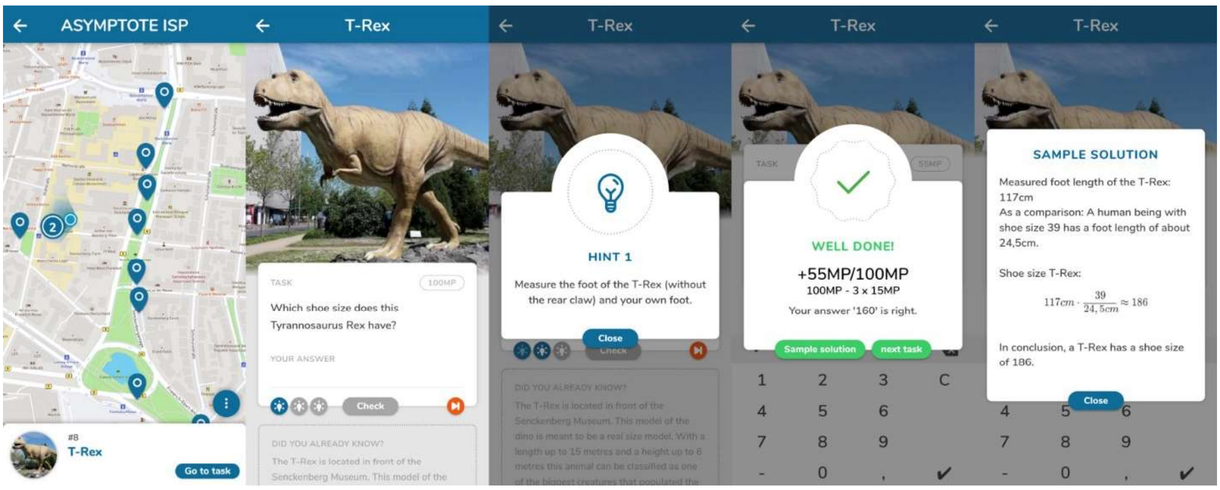

Figure 2: Screenshots from the MathCityMap app (a: Navigation, b: Task Formulation, c: Hint, d: Solution Validation, e: Sample Solution).

For mathematics trails in the educational purpose, a web portal for creating tasks and a smartphone app for walking mathematics trails are available. The smartphone app supports the students while they walk along a mathematical trail previously created in the portal and compensates the teacher's absence with its features when the students work outside the classroom. To do so, it shows the students' position and the location of the tasks (Figure 2a). A picture of the task object is available for clear identification. Furthermore, the tasks previously formulated by the teacher are displayed (Figure 2b), including up to three formulated hints (Figure 2c). In addition, the app provides direct feedback on the entered solution (Figure 2d) and displays a sample solution (Figure 2e). Since the Covid-19 pandemic, the scope of MathCityMap has been extended from outdoor mathematics to online education. During the Emergency Remote Teaching phase, it was used to compensate the absence of the teacher for distance education (cf. Barlovits et al., 2021) and therefore used in different contexts besides outdoor learning.

Independent of the context, two theoretical potentials of the MathCityMap features for mathematical modelling emerge (cf. Ludwig & Jablonski, 2021; Jablonski, 2023): On the one hand, the hints offer the possibility - depending on the formulation - to initiate and support individual steps of the modelling cycle. For example, support for structuring and simplifying the mathematical problem or help for mathematizing and the mathematical work can be implemented by the teacher through the hints feature beforehand. On the other hand, the validation option as described before, offers the possibility to initiate this step which can only rarely be found in the modelling activities. Especially in the area of modelling tasks, the solution check is conducted via an interval, which is determined in advance by various modelling steps and calculations and anticipating any deviations that could realistically occur. This kind of corrective feedback might draw students' attention to the fact that a new run of the modelling cycle might be necessary.

With these potentials only being on a theoretical level, the article aims at an empirical proof of the potentials of MathCityMap in mathematical modelling activities. Therefore, the research question *What is the role of the digital tool MathCityMap in students' modelling activities?*

METHODOLOGY

To answer the research question, the data from a qualitative study conducted in 2022 are partly taken for this article (for the whole study see Jablonski, 2023). In the total study, the different contexts of mathematical modelling were compared. For the research question of this article, only the sample with the focus on the digital tool MathCityMap is relevant. It comprises 10 students visiting the enrichment program *Junge Mathe-Adler Frankfurt* in grades 6-8. The students were divided into three groups. Each group worked on three tasks related to geometric objects (cf. Figure 3; from left to right): The height of the Body of Knowledge sculpture, the volume of the stone and the surface of the Rotazione sculpture.

One of the tasks was to determine the size of the *Body of Knowledge* sculpture if it were standing. The sculpture represents a seated person with legs drawn up. In order to provide a holistic view on mathematical modelling and its different contexts, three task settings were defined for each of the objects:

● Outside at the real object: The students solved the task outside directly at the real object. They had measuring materials with them as aids (see Figure 4a).

● Inside with photos: The students solved the task using a series of photos of the real object with a person as a possible reference size (see Figure 4b). In addition to the photos, they had a set square at their disposal for measuring sizes.

● Inside with 3D printing: To solve the task, the students were given a 3D representation of the real object, which had previously been printed to scale (see Figure 4c). A LEGO figure and a set square were provided as a reference size.

Figure 3: The task objects (a: Body of Knowledge, b: Stone, c: Sculpture).

Figure 4: Different Representations of the Body of Knowledge (a: Real Object, b: Photo, c: 3D Print).

The settings and objects were arranged systematically according to a Latin Square Design: Group A solved the Body of Knowledge with the 3D Print, the Stone at the Real Object and the Sculpture with Photos. Group B in comparison solved the Body of Knowledge at the Real Object, the Stone with Photos and the Sculpture with the 3D Print. Finally, Group C solved the Body of Knowledge with Photos, the Stone with the 3D Print and the Sculpture at the Real Object.

While working on the tasks, the groups had a smartphone, which they used to access the tasks via the MathCityMap app. Hereby, they were able to use hints that should support the *Simplify and Structure* and *Mathematize* steps, e.g.,

- Hint 1: Look for a model that you can use to describe the Body of Knowledge. There are several models that could be considered and none will fit perfectly;
- Hint 2: You can work with proportions, for example;
- Hint 3: Think about the values you need for your model and how you can approximate them as accurately as possible.

While using the hints was optional for them, the groups were explicitly asked to have their result validated by the app. After solving a task correctly or giving up, the students were able to view the sample solution with a possible solution of the modelling task.

During the processing of all tasks, the groups were accompanied by a student assistant who filmed the interactions. As the student assistant gave not content-related advice, the situation resembled a modelling process without the teacher being present as it happens during a math trail or in distance education. The video interactions were transcribed, coded deductively using the modelling steps according to Blum and Leiss (2007) despite Presenting (which was not relevant in the material) and visualized with activity diagrams (Ärlebäck & Albarracín, 2019). Besides, a qualitative content analysis according to Mayring (2000) was used to extract the main activities being related to the MathCityMap app. In addition to the video recordings and the written records of these groups, the app activities of the groups were recorded using MathCityMap's Digital Classroom. In an exported Excel list, the group's events (retrieval of hints, solution input) were made available with a timestamp so that they can be directly associated with the video recordings. Accordingly, MathCityMap events were labeled in the representation of modelling activities.

In a questionnaire that was given to the students after each modelling task, five items related to hint use and feedback on the solution provided by the app were given. The students were asked to give their agreement to the five items on a scale from *0: Not agree* at all to *4: Totally agree*. Since hint use in particular was optional and incorrect feedback also did not occur for all groups, *Not Used* was added to the scale.

- MCM_1: The hints helped me to structure the solution process.
- MCM_2: The hints helped me identify what math I needed to solve the problem.
- MCM_3: The feedback on the result allowed me to evaluate my solution as correct/wrong.
- MCM_4: In case of wrong feedback, the app made me check and improve my solution.
- MCM_5: I would like to solve tasks with the app more often.

RESULTS

The three groups took 21 hints. A total of 39 solution inputs were validated for the nine tasks. Finally, six tasks were validated as correct using the previously determined solution interval. In addition, the sample solution was consulted after four modelling processes – once after a correct solution, three times after multiple incorrect entries.

Figure 5 shows the activity diagram of one group solving the three tasks in the different contexts. The analysis of the diagram is done as an example for the task that a group worked on inside the classroom with photographs (second task). The group started to understand the modelling task, continued with structuring and simplifying the real situation and mathematized the real model. After a short period of mathematical work, the group entered their first result in the app which was validated as wrong. Based hereon, the students continued to validate their result and procedure and started a second attempt to solve the task properly. They rethought the steps of mathematizing, structuring and simplifying and working mathematically. Afterwards, they checked a second result in the app which was again rejected. Afterwards, they took the first hint and managed to proceed the task with a correct result through validation and identifying a previously made mistake.

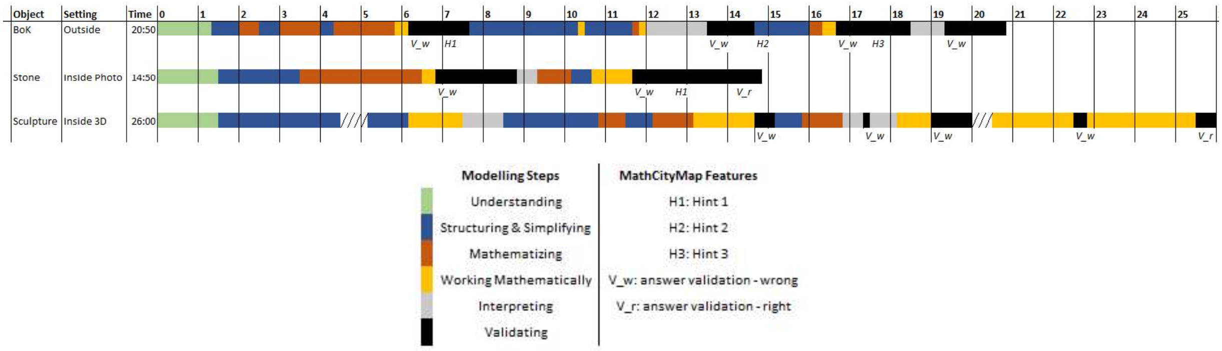

Figure 5: Example of an Activity Diagram visualizing a Modelling Process.

With respect to hint usage, the following observations can be formulated from this and the other two groups' diagrams: The hints were generally used in the course of an *on-going* modelling process – for most groups, the hints were only retrieved after an initial solution had been evaluated as "wrong" by the app. Thus, for this sample, the hints did not serve too much to initiate specific modelling steps, but to re-discuss and revise them. Based on the qualitative content analysis of the group interactions, the following support can be described in relation to the modelling steps:

- Simplify and Structure (2): In particular, hint 1 guided the modelling step and led to (re)discussing the choice of a real model considering inaccuracies.
- Mathematize (3): Through hints 2 and 3, aspects of mathematizing were addressed. Since these hints were taken only after prior mathematizing, they led in this phase to reconsidering and, if necessary, adjusting the data selection.
- Mathematical Work (4): In comparison, the hints were rarely used in the step of mathematical working. Since the hints did not directly fit to this step due to their wording, the students did not directly refer to the hints in their further work.
- Validation (6): The hints were used immediately after the app validation to assess why the entered result was evaluated as "wrong" (see the example in Figure 5).

The solution validation by the app was in all cases directly related to the modelling step of validation (6). In one case, a solution was validated as correct directly after the first input, which did not lead to any further action on the part of the students. In the other eight modelling processes, a solution was validated as "incorrect" at least once. After the first incorrect entry, the feedback from the app led all groups to start a second run of the modelling cycle – usually with the Simplify and Structure or Mathematize step. If there was another incorrect entry, sometimes a third attempt was started, in which the students checked their steps and changed them if necessary. If, despite repeated attempts, the app still validated the solution as "wrong," the sample solution was used in this case to complete the validation step. In these cases, the groups used the sample solution to compare their approach to an exemplar model and to identify potential problems.

For the questionnaire items, the following data can be reported: The students perceive the hints as rarely useful ($M_{MCM\_1} = 1.2$; $M_{MCM\_2} = 1.1$). In contrast, the evaluation tool of the app was perceived more positively ($M_{MCM\_3} = 3.1$; $M_{MCM\_4} = 2.5$). The wish for future use of the MathCityMap app reaches a mean value of 2.8.

DISCUSSION

With the presented results, it is possible to reflect on the question of MathCityMap's role in students' modelling activities. The students of this sample used all three features provided by the app: hints, solution validation and sample solution. The hints were mainly not used in the originally intended way: Whereas the hints should initiate modelling steps, the students mainly searched for support only after the first run through the modelling cycle and a negative solution validation. This is also reflected in the students' perception of the hints in terms of their guidance and usefulness. Therefore, it makes sense to formulate the hints in such a way that they do not initiate but support the modelling steps precisely, e.g. proposing useful simplifications or appropriate models.

For the solution feedback, it can be concluded that this feature helps students in validating their work – a step that usually does not take place independently on the side of the students. Through the app validation, the students started a new attempt to solve the task and revised problematic modelling steps from their first trial. This observation can also be linked to the agreement of the students to the corresponding items. For the sample solution, potential can be seen in terms of a complete and explaining feedback instrument. Whereas the automatic feedback can be valued as a meaningful feature in the context of the modelling cycle, the sample solutions play a comparable minor role. It was only used after inserting multiple wrong results. It shows that the students completed the task with the app giving a "correct" feedback and did not show additional interest in different modelling procedures. Therefore, the app can serve as a feedback instrument, but should not replace the discussion of different modelling strategies thereafter.

Concerning the different modelling contexts, the made observations show no relevant differences in the app usage dependent on the context. The groups used the app similarly in the different contexts. The predominantly positive attitude to future uses of the app, allows the proposal to include corrective feedback through digital tools in mathematical modelling activities as it happens in MathCityMap for outdoor education and modelling contexts inside the classroom. Even with their teacher being absent, the digital tool can help students to understand the modelling process as a process that has to be revised and improved before a result is acceptable. In addition, it helps them to understand their result as valid and might lead to more independent validation activities in future modelling activities. This hypothesis as well as the potential of improved and differentiated hints are to be examined in future research and in research with a special focus on digital tool in distance modelling activities.

In this future research, two limitations of the presented study should be considered in more detail. The first is the choice of sample, i.e. gifted and interested students. It might be that the results need adaptations when being transferred to more heterogeneous groups. Mathematically gifted children might bring special (problem solving) skills that could support their work in the modelling tasks as well. In addition, the sample of this study should be extended in terms of the quantity of observed students.

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TEACHING COMPUTATIONAL THINKING WITH <COLETTE/>

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Abstract. Computational Thinking (CT) is a necessary skillset to navigate and participate in our digitalized world. It is, thus, imperative to teach this skillset in school. Because CT is not automatically acquired simply by using digital tools, but it needs to be taught deliberately. However, a lack of easy-to-use teaching material and teacher training has been shown to hinder the adoption of CT-specific education in the classroom. Here we show a low-threshold approach to teaching CT using the <colette/> web portal and app. The web portal provides a convenient way for teachers to create tasks and the app allows students to display, solve, and review the tasks. Teachers create custom paths by adapting the predefined task family templates, each addressing a specific CT-skill. In this way, <colette/> facilitates teaching CT and integrating it into various school subjects.

Key words: Computational thinking, digital teaching, generic tasks.

INTRODUCTION

The ongoing digitalization of modern society makes it necessary to be skilled in using computers daily in various situations, for example in the workplace, but also increasingly for mundane matters. Surprisingly, the skills needed to employ computers (and computing devices) efficiently are not acquired by solely using them without guidance but need to be taught deliberately. In 2006 Wing coined the term Computational Thinking (CT) meaning this skillset needed in a digitalized world.

"Computational Thinking represents a universally applicable attitude and skill set everyone, not just computer scientists, would be eager to learn and use." (Wing, 2006) At its core, this attitude or skillset is independent from technology and has been debated which skills or competencies are related to it and if they are disjoint from each other. Bocconi et al. (2016) have worked through numerous papers and found that the skillset most referred to as CT consists of: Abstraction, Decomposition, Generalization, Algorithmic Thinking, Automation, and Debugging.

Abstraction is the skill used to see a real-life object and discard unnecessary details to then replace this real-world object with a fitting model. Even the best fitting model might make it necessary to further decompose (Decomposition) the given problem and maybe one recognizes patterns in similar problems to then find a general way of solving them (Generalization). This process of solving the problem can involve clearly defined steps (Algorithmic Thinking) which are ideally understandable not only by humans but also by machines. If there is an algorithm which is understandable by a machine, we can exploit that fact by using the machine e.g., for repetitive tasks (Automation). Lastly, with the skill Debugging one can find errors in a given code as well as understand how an algorithm works by mentally going through each of its steps.

This CT skillset is not only necessary to deal with computers but it's a set of skills that can be generally applied to solving problems. All of these skills are not only skills necessary if one deals with computers, but they are a set of generally applicable skills while solving problems.

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(Wing, 2006). As CT-skills are not only applicable for coding, it is possible to integrate them also in other school subjects than Computer Science. Thus, CT is most of the times integrated in the curricula in one of the three following ways (Bocconi et al., 2016): As a cross-curricular theme, as part of a separate subject or within other subjects. For all the three possibilities there is a necessity for qualified teachers and for easy-to-use, inexpensive material to teach CT (van Borkulo et al., 2020).

In this paper, we show how the <colette/> app and web portal will allow teachers to easily integrate CT-specific contents into their lessons. As automated learning environment, <colette/> will be pre-populated with various Task Families (acting as blueprints for the specific tasks) and teacher training materials and it allows students to independently solve and review the chosen tasks. In what follows, we first introduce the <colette/> project before showcasing the workflows of creating CT-specific tasks via the web portal and solving and reviewing the tasks in the smartphone app. Afterwards, we address the general concept of Task Families, and discuss the specific ideas for the two already available Task Families.

TEACHING COMPUTATIONAL THINKING VIA THE <COLETTE/> WEB PORTAL AND APP

Project Goals of <colette/>

The research-project <colette/> conducted by a consortium of six partners all across Europe (Germany, Austria, the Netherlands, France, Slovakia). The aim of the project is to create a web portal, an app, didactical content and a short-term curriculum for teacher trainings, in order to facilitate integrating CT into preexisting lessons. In addition, serving as authoring tool, the web portal includes teacher material such as introductions to CT and guides on how to approach the topic in class, while the smartphone app gives the students direct access to material, hints and the feedback using their own smartphone ("bring your own device"-approach).

Creation of Paths with Tasks in the Web Portal.

To visualize how <colette/> allows teachers to integrate CT into lessons, Figure 1 shows a screenshot of the <colette/> web portal, as it appears when creating a new learning path (Roth, 2015). In an initial step, the teacher assigns the path a name (e.g. "Fun with Drones"), sets the target group (e.g. "upper secondary") and gives a description. Afterwards teachers can start adding various tasks to the path.

Adding tasks starts by selecting a task family (Figure 1, step 1), in this example the screenshots are taken from the Task Family *Building Cubes*. To date, we have made available two different task families, *Building Cubes* and the *Drone* (for further details see *Task Family: Building Cubes* and *Task Family: Drone*), with each task family acting as a sort of blueprint for the creation of specific tasks. In a later section, we'll expand further on the concept of task families, and present the two available task families in detail. After choosing a task family, the teacher is guided through the creation of a specific task. In the example of Figure 1, the teacher chose the task family *Building Cube,* and selected *Implementation* as assignment type ("*What should the student do?")* (Figure 1, step 2). In this assignment type, the student will create an algorithm to place cubes on a 3D grid to create a desired shape, the result will be visualized in augmented reality (AR). In step 3 the teacher picks which shape will be implemented. In case of Figure 1 it is a *square pyramid*.

The next step is defining the settings of the pyramid. Here the teacher can choose the height of the pyramid and the location by defining a starting block through given (x,y,z) coordinates (Figure 1, step 4). The right-hand side of Figure 1 (step 4) shows a preview of the defined pyramid. Finally, step 5 allows to take a screenshot of the preview, change the problem definition and adjust or delete the hints. When the teacher completed all steps, they then can add the task to the path.

Once all tasks are created a 5-digit alphanumerical code is generated that identifies the path and can be shared with students to enter in the app.

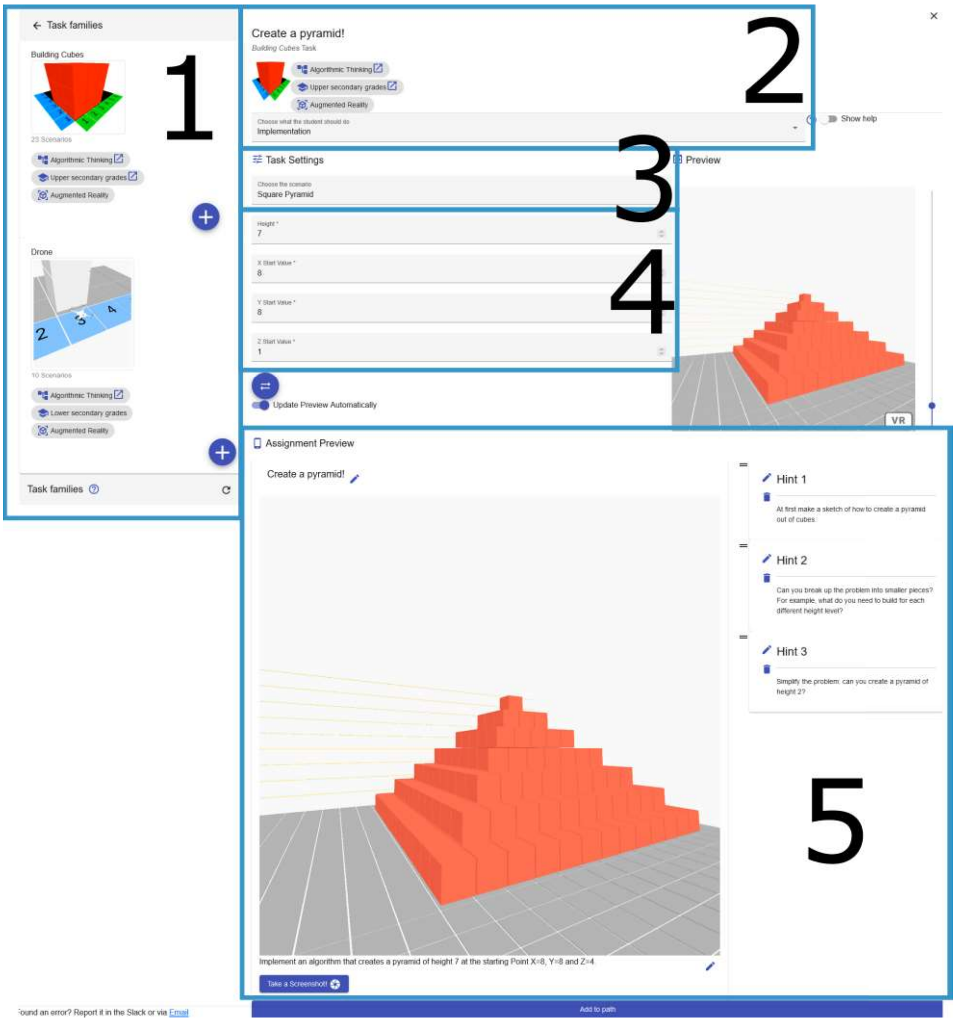

Figure 7: Creating a task in the <colette/> web portal (screenshot).

Figure 2 shows the workflow of a teacher in an abstract way. The teacher starts by selecting a Task Family and an assignment type (“*What should the student do?”)* and then chooses a scenario and final settings. The problem definition, picture and hints can be edited as well. Note how the teacher’s choice of the *“What should the student do”* (assignment type) corresponds to a specific *solution verification process* that the portal will employ. The solution verification process allows the portal to automatically verify a given solution. After the settings have been set the sample solution and the expected result have been created. The expected result is in Figure 1 the actual pyramid structure.

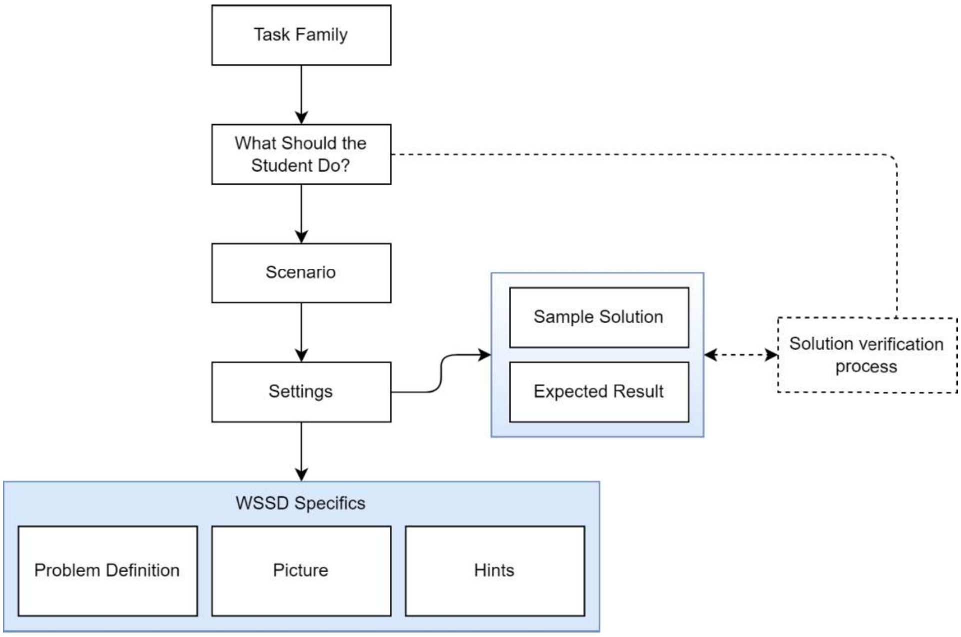

Figure 8: This flow chart shows the process of creating a task in the <colette/> web portal.

Viewing and Solving a Path in the <colette/> App

Once a learning path has been created by the teacher, the alphanumeric code of this path will be shared with the students. After starting the smartphone app, any previously added paths are visible in a list. New paths are added by tapping the small plus sign on the upper right-hand side of the screen (see Figure 3a). Entering the path code into the shown dialog (Figure 3b), the students can view the paths general information (Figure 3c) and start the path by clicking the start button in the lower part of the screen, in Figure 3 the student will work on a task from the Task Family *Drone*. The blue retry-button at the right resets progress in this path. The red “remove” button (shown by a -) on the left-hand side of the paths symbol removes the path from the list of saved paths (Figure 3c). If this was chosen the path can be readded with the code.

In Figure 3d we see a list of all the tasks in the path, the first one being the only one available to us at this moment. The others are not yet unlocked but can be started once all previous tasks are solved or skipped. Figure 3 goes on to show a task from the *“Drone”* task family, where students use block-based programming for the flight path of an augmented reality (AR) drone. Tapping on one task opens the screen seen in Figure 3e where we can see a row of categories for block-based programming expressions at the top. These can be opened and the specific expressions dragged down into the *Blockly* workspace (cf. Weintrop, 2019). Blocks that are no longer needed can be dragged into the trash bin in the lower right-hand side of the workspace to be removed. Below is the assignment given by the creator of the task, that can be collapsed to expand the workspace.

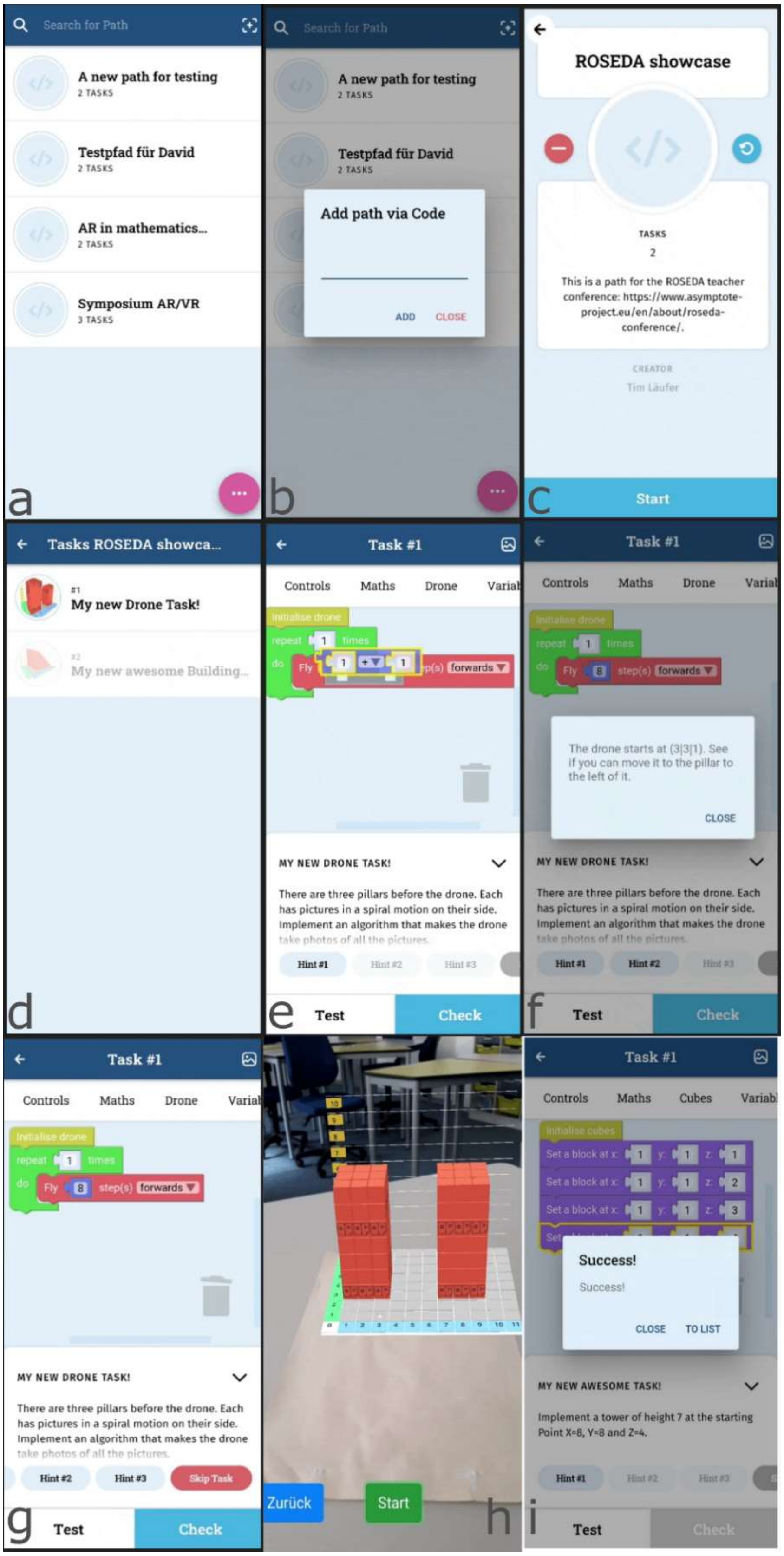

Figure 9: Viewing, solving and reviewing a task with the <colette/> smartphone app (screenshots).

The tiered hints can be tapped to open them (Figure 3f). When all hints have been consecutively opened the task can be skipped in case a student has trouble solving this task (Figure 3g). The button “Test” opens the AR preview (Figure 3h), where the solution of the programmed algorithm is displayed. There students can validate their answers before tapping the button “Check” to have the solution verified by the app (Figure 3i).

TASK FAMILIES AS EXTENDED GENERIC TASKS

Having seen the workflows of creating and solving a task, we now address the underlying concept of task families. In an earlier publication (Milicic et al., 2020), we introduced the concept of “generic tasks”, meaning a simple structure for creating up to five different tasks from a common set of algorithm, wanted outcome and a description. By combining only those three components one can easily (and in automated fashion) derive five different tasks: implementing or analyzing a code, finding the error in a given code solving Parson’s problem, as well as Parson’s problem with distractors. A Generic Task itself, thus, cannot be assigned to a student. It is rather a building plan of a specific task.

Despite the variances in the task design, those Generic Tasks facilitate the process to provide material for different CT-skills as well. In our previous study, 14 experts concluded that these Generic Tasks address abstraction, algorithmic thinking, automation and debugging, thus supporting four out of the six CT-skills.

The concept of the Task Families generalizes the idea of the Generic Tasks. Task Families are a blueprint with which one can create several Generic Tasks which all have some settings in common. This is the foundation for the task editing process in the <colette/> web portal. We now discuss the two already available task families *Building Cubes* and *Drone* and show how Task Families expand the concept of Generic Tasks.

Task Family: Building Cubes

The Task Family *Building Cubes* allows the creation of tasks that revolve around 3D-shapes built on an augmented reality (AR) checkerboard, that can be viewed through the smartphone app directly on the students’ desk using the AR marker. The shapes consist of individual cubes that are placed in their position via an algorithm that is editable with the visual code *Blockly.* For larger and more complex shapes, this building algorithm typically requires various loops and conditions.

From the perspective of Generic Tasks, a given shape (e.g. a pyramid), the building algorithm, and the problem definition can be seen as the three ingredients from which the five generic tasks can be derived. Currently, <colette/> provides the assignment type *Implementation*, in which students are asked to create an algorithm, and the existing algorithm is only used in the back-end to automatically verify the solution. In further iterations, we’ll provide the other Generic Tasks (analyzing the code, debugging, Parson’s problem).

The Task Family extends this concept of Generic Tasks by giving the choice of multiple figures to create. Teachers can choose between a cube, a table, a car and more. This means that the Task Family is a collection of Generic Tasks.

The assignment type *Implementation* addresses the CT-skill *Algorithmic Thinking,* while other assignment types will address other CT-skills (Milicic et al., 2020). Apart from CT-specific skills, we note that working with the augmented reality (AR) environment also addresses spatial recognition skills (Boon, 2003). Evidently, 2-dimensional screens (or paper) can only show 3D-objects as 2D-projections. Mentally translating 2D- into 3D-shapes (and vice versa) is a skill that needs to be trained and viewing and interacting with the shapes in the AR-view helps develop it. A student can walk around the programmed object (by walking around the AR-marker) and looking at it more closely and see it as if it were on their desk.

Task Family: Drone

The task family *Drone* is like *Building Cubes* in that it works with 3D-shapes placed on an AR-checkerboard, and the algorithm of interest is editable via *Blockly*. However, in the *Drone* task family, the algorithm is not used to create the 3D-shape (which is fixed here), but rather to direct the flight of a (virtual) drone, and have it perform tasks such as taking pictures from the viewpoint of a certain position.

The 3D-shapes are set up by the web portal in a generator, with available shapes comprising structures such as a wall with a window. In the previously shown example (Figure 3h) two pillars are placed on the checkerboard, and markers are placed in an upward spiraling pattern on the sides of those pillars. In the assignment type *Implementation*, the student programs the flight path of the drone in such a way that all the markers are being photographed subsequently by the drone.

Importantly, the movement of the drone is directly displayed in the AR view as animation, as well as the photos it takes. In comparison to the *Building Cubes*, we thus expect the *Implementation* task of the *Drone* task family to have a stronger process-character, because the students can investigate their algorithm sequentially and localize the error while the drone performs the actions. Therefore, *Debugging* is strongly addressed, as well as *Algorithmic Thinking*.

CONCLUSION AND OUTLOOK

In this paper, we presented our vision for <colette/>, an automated learning environment designed to facilitate teaching CT in class. We showed how teachers can use the web portal to create tasks by customizing a set of predefined Task Families that specifically target CT-skills, and how students can use the smartphone app to view, solve and review these tasks. We discussed how the concept of Task Families allows to (automatically) create variations of tasks from a common set of parameters, helping teachers to create effective material quickliy.

In addition to the two presented Task Families *Building Cubes* and *Drone*, we have designed more Task Families that will be available in the public release of <colette/> (first public release scheduled for 2023). Additional Task Families are designed for unplugged lessons and will be available in the handbook of <colette/>.

We have presented the assignment type *Implementation*, in which students themselves create the code of interest with *Blockly.* While this assignment type mostly addresses the CT-skill *Algorithmic Thinking*, the public release of <colette/> will make available more assignment types (thus following the concept of Generic Tasks) and thereby addressing a whole spectrum of CT-skills.

With the addition of more Task Families, we will also emphasize that <colette/> is not exclusively designed for Mathematics and Computer Science classes but can be integrated in various school subjects. With this, we aim to reflect the general applicability of CT-skills, which are essential to navigate, participate, and understand an increasingly digitalized world.

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TOPIC 3

RESEARCH ON TEACHING AND LEARNING DURING THE COVID-19 PANDEMIC

THE POSITIVE SIDE OF THE PANDEMIC, DOES IT REALLY EXIST?

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Abstract. The immediate shift from face-to-face to distance instruction caused by the Covid-19 pandemic has raised teachers' awareness of the significant role that digital tools can play in educational processes. This shift forced teachers to use different kinds of digital technologies in various ways to overcome the challenges they encountered as a result of the pandemic. In this keynote talk, I will discuss how teachers use digital tools for educational proposes, aiming to extract insight regarding how such tools can be used beyond the Covid-19 period. In addition, I will present innovative approaches towards using digital tools, the need for which arose during the lockdown period, and how these approaches can be integrated into ordinary educational sessions at schools.

Keywords: Digital technology, learning and teaching mathematics, post pandemic.

INTRODUCTION

With the outbreak of the global Covid-19 pandemic in 2019, many workplaces began working under emergency regulations. Schools across the world closed their gates and transitioned, without prior warning, to distance learning and instruction. Many teachers found themselves in a different reality than they were used to. The new reality forced them to change their work practices and deal with difficulties and challenges that they had not experienced before. Teachers found themselves obliged to improvise teaching methods and use new methods to maintain proper teaching practices. Due to the closures and restriction of social interaction, teachers and students had to use various ubiquitous digital technologies (e.g., email, video clips, WhatsApp, Zoom, MS Team, etc.) to communicate with the students and to continue the instruction process. I will point out that these technological tools are not necessarily the strong side of all teachers, and the degree of skill in using these tools differs from teacher to teacher.

This new reality invited many researchers from all over the world to explore the teaching, learning, and evaluation processes that emerged during the Covid-19 period (e.g., Adedoyin & Soykan, 2023; Cusi et al., 2022; Thurm et al., 2023). Many studies have focused on the challenges teachers face as they transition from classroom to remote teaching, looking at how teachers have tackled these challenges and how technological tools have helped them overcome these challenges (e.g., Aldon et al., 2020). In my talk, I will begin by discussing and mentioning some of the challenges that teachers have faced. Second, I will present some methods that teachers have employed to overcome these challenges. In the third part of my talk, I will introduce learning systems that have been developed or are still in the development process, which are endeavoring to overcome challenges that arose during the Covid period. In particular, I will introduce an AI-based learning system that aims to track students' learning processes in real-time, and to inform teachers of prominent characteristics in the learning process or so-called critical moments during learning (Segal et al., 2017). Information about these processes is especially important for teachers who must know what is happening with students who are physically far from away. The second system, called MathematicX, allows the teachers to determine the contents of the lesson, and

Swidan, O. (2023). The Positive Side of the Pandemic, Does it Really Exist?. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 133–143). WTM. https://doi.org/10.37626/GA9783959872522.0.16

then leads the students in a differential way toward achieving the pedagogical goal set by the teacher. The third system, Dash4Emotion, is a dashboard-based system that tracks a change in students' sentiments and alerts the teacher about the students' emotional status. This system is important because the emotional aspect plays a significant role in a crisis learning period.

CHALLENGES IN THE PROCESS OF LEARNING MATHEMATICS DURING COVID-19

As I mentioned above, the Covid-19crisis surprised educational systems, revealing that some of the teachers were not qualified enough to deal with the demands of teaching under the conditions required by this period. Teachers began to teach in a new reality, accompanied by many challenges. The teachers dealt with the challenges, sometimes with the help of the educational institutes and sometimes alone, and offered solutions to overcome these challenges. The research literature has discussed several challenges, and I have no intention of reviewing them all in this paper. On the contrary, I will focus here on the central themes of challenges that arose from the research literature in order to establish a robust basis that allows me to discuss the role of the innovative technologies that were inspired by or greatly developed during the Covid-19 period. To this end, in my talk, I will focus on several central themes of challenges that have been discussed in Aldon et al.'s (2020) paper: The challenges associated with limiting social learning and applying contemporary teaching methods, challenges related to supporting students who face difficulties, challenges related to identifying student emotions, and challenges related to student assessment.

One of the main challenges teachers met during the Covid-19 period was the limitation of applying collaborative learning, such as learning in small groups, and implementing contemporary teaching methods, such as inquiry-based learning (Aldon et al. 2020). Although online learning platforms, like Zoom or MS teams, allow breakout rooms, the teachers found it challenging to manage collaborative learning. They attribute this challenge to the fact that the teachers cannot be involved in more than one breakout room at the same time. The presence of the teacher in one room for a while prevents him/her from seeing what is going on in the other rooms. Unlike the classroom setting, where the teacher has more control and is more aware of what is going on in several groups simultaneously, online platforms make supervision more challenging, or at least teachers were not practiced in conducting it through this medium.

The shift from face-to-face to remote learning has limited the implementation of contemporary teaching approaches, such as inquiry-based learning, while strengthening the transmission approach of teaching (Aldon et al. 2020). One possible reason for this challenge is the limitation of teachers' ability to guide students' exploration. In this context, research has found that the guiding role of the teacher in supporting students is essential to the effectiveness of inquiry-based learning (Lazonder & Harmsen, 2016). In fact, if inquiry-based learning is interpreted as an approach in which the "learner is not provided with the target information or conceptual understanding and must find it independently and with only the provided materials" (Alfieri et al., 2011, p. 4)—in other words, as learning in which the teacher has no significant role— it has limited educational value (Scott et al., 2018). To overcome this challenge, teachers found themselves adopting a transmission approach to learning, though they were not fully convinced of its effectiveness (Aldon et al, 2020).

A more serious challenge than limiting the application of contemporary teaching methods is the teachers' feeling that they are not engaging in a significant learning process when learning takes place remotely. This is actually a very significant challenge, with teachers feeling they have had to settle for maintaining what students have learned in the past (Aldon et al., 2020). Apparently, due to the inability of some teachers to take advantage of the digital tools available to them or to instrumentally orchestrate the digital tools (Drijvers et al., 2010), they were content with teaching through lecturing. This way of instruction led to a lack of interaction between teachers and students, which resulted in students' boredom and lack of motivation for learning.

Identifying student emotions and regulating them during the teaching process is significant for the success of the learning process. If this claim has been confirmed in ordinary days, it holds even more true in times of crisis, when feelings like fear, uncertainty, frustration, and boredom are common. In distance learning, even if the students' cameras are turned on, identifying students' emotions during the learning process poses a significant challenge. This is one of the findings that has been revealed in studies that examined the challenges teachers met during the Covid-19 period (Aldon, et al., 2020). Teachers, partly due to technical limitations, found it difficult to characterize the emotions of the students during remote learning, and they felt powerless to help students regulate their sentiments to convert them from emotions that hindered learning to emotions that advanced learning.

In a democratic society, educational systems should provide an equal opportunity for every student to learn. This goal is even more critical during a pandemic period, where social and cultural differences can easily manifest. Supporting low-achieving students and students with difficulties was found to be an essential challenge encountered by teachers. More precisely, to help students who face difficulties, teachers were asked to provide materials that help students to follow the lessons at their personal rate. In addition, they were asked to devote time, often after hours, to answer students' questions and to re-explain topics. These are only two examples, among many others, that illustrate the compensatory actions teachers were required to take in order help students overcome the difficulties they face during distance learning.

Assessment, in addition to learning and teaching, is another component of the educational activity. In fact, assessment is always a fundamental concern for teachers. On the continuum between summative and formative assessment, teachers, based on their beliefs and learning assumptions, implement strategies to make decisions about their future teaching (Perrenoud, 1998). The aim of summative assessment is to evaluate student learning at the end of an instructional activity by comparing it against some standard. The aim of formative assessment is to monitor student learning to provide ongoing feedback that teachers can use to improve their practices and students can use to improve their learning. Formative assessment is different from other kinds of assessments in that it is used in the process of learning, not at the end of it. The formative assessment focuses on students' learning processes, collected during teaching and learning activities, to make "decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7).

Implementing both kinds of assessment has been found challenging for many teachers. Cusi et al. (2022) found that one of the main challenges teachers met during the Covid-19 period was related to different technical problems that prevented them from effectively activating typical formative assessment strategies, such as designing and conducting whole classroom discussions aimed at eliciting evidence of students' understanding. Another challenge associated with formative assessment is related to specific difficulties faced by teachers in following students' processes due to the difficulty of activating multimodal communication. Concerning the summative assessment, Cusi et al. (2022) found that the need to ensure the reliability of the exams and of being sure that students do the exams by themselves, without external help, is one of the main challenges teachers meet. The second main challenge that Cusi et al. (2022) found was related to teachers' difficulties in identifying the right objects of assessment during the Covid-19 period. The teachers, especially those whose students were involved in national final examinations, expressed their worries due to the lack of clarity about the mathematical topics involved in the final examinations and the ways of assessing them.

DIGITAL TOOLS USED BY TEACHERS TO OVERCOME THE CHALLENGES

The commonly used digital technologies (email, video clips, WhatsApp, etc.) played a key role in helping teachers overcome the instructional challenges they faced during the Covid-19 period. To support the transmission approaches to learning, teachers sent the learning materials to their students via email. Others video recorded themselves solving exercises or explaining a new topic and posted their clips via the YouTube platform. A third group of teachers used instant messaging applications to support their students and to help them be engaged in the learning process. In addition, teachers used several video communication platforms (i.e., Zoom, MS Teams, and others) to manage the learning process. They utilized the affordance of these platforms to transmit knowledge and to share mathematical topics with their students.

Although teachers employed common digital technologies when they implemented contemporary teaching approaches, such as inquiry-based and collaborative learning, they found this use challenging, as I described above. The teachers used Dynamic Geometry Environments (DGE) (e.g., GeoGebra) to engage their students in inquiry-based learning. The teachers were asked to prepare and provide their students with appropriate tasks to prompt the inquiry processes. The software and the appropriate tasks are two essential elements for inquiry-based learning to take place, but they are not sufficient. In fact, the commonly used digital technologies do not have a feature that can provide teachers with the information needed for prompting inquiry-based learning. To prompt inquiry-based learning, teachers have to be aware of the students' learning processes in real-time to guide their exploration processes (Lazonder & Harmsen, 2016; Schwarz et al., 2021).

In the same line of thought, prompting collaborative learning and fostering argumentation-based learning is not only an issue of setting the students together to discuss or solve mathematical problems or exercises. On the contrary, teachers have a crucial role in these learning settings. To ensure the evolution of the learning process, they should mediate the daily concepts that may emerge in the students' discourse, guiding them towards the scientific concepts that are culturally accepted among the scientific community (Vygotsky, 2012). Also, teachers should ask open-ended questions to support students in argumentation and encourage dialogic interactions between students (McNeill & Pimentel, 2010). Thus, teachers who wish to advance the collaborative and argumentative learning process must be aware of the students' learning progression and the content of the discourse.

To benefit teachers in remote learning, digital tools must provide the teachers with the information and the data they need. To my knowledge, the ubiquitous digital technologies commonly used during the Covid-19 period did not fully meet this requirement. To fill this lacuna, teachers used video recording to implement teaching methods similar to the flipped classroom approach (Tang, et al., 2020). In doing so, teachers prepared short-duration clips that introduced the concepts to the students. Students were asked to watch the video before the lesson. During the lesson, the teacher interacted and engaged in discussion with the students based on what they watched (Aldon et al., 2020).

To conduct collaborative learning sessions, teachers utilized the breakout room options available in several video conference platforms (e.g., Zoom, MS Teams). The creation of small groups allowed the exchange of ideas between the students. The teachers, who attended the virtual room from time to time, could offer guidance only based on what they observed or heard through their presence in the virtual room. Other teachers asked their students, in groups, to prepare a summary of the lesson and to show and explain their summary during the lesson session (Aldon et al., 2020).

Teachers used several kinds of technologies to support students who face difficulties during their learning process. The most common tool teachers used to support their students was instant messaging technology. For example, teachers used the option in MS Teams to send personalized messages to their students to find out how the exercises were going and to offer them their support. Other teachers used one-to-one communication applications such as WhatsApp or messenger to offer private help to the students. A third group of teachers used video recording to provide explanations targeted to individuals. Some teachers grouped lower-achieving with high-achieving students, hoping the latter would support the former.

Identifying and dealing with the students' emotions was one of the main challenges teachers faced during the Covid-19 period. To overcome these challenges, teachers connected with their students daily via instant messaging tools or emails. Teachers used, for example, WhatsApp recording options to ask about the student's emotions and feelings (Aldon et al., 2020). Some used the chat option, available on the video conference platforms, to ask students individually about their feelings during the lesson. A third group of teachers chose to contact their students directly via phone call if the teacher recognized the need to do so. Although these digital tools are useful for supporting students emotionally, they are limited. These tools allow the teacher to support the student after the lesson is finished or to support students when the teacher recognizes that they need affective help. However, these tools are not useful when teachers want to support several students at the same time or cannot help in supporting students that the teachers do not recognize as somebody needing help.

Regarding the assessment issue, teachers used several platforms and strategies to assess students formatively and summatively. They used platforms that automatically provided grades, or they adopted non-educational software, such as Google forum, to assist them in collecting data and organizing the results. The teachers also implemented several kinds of strategies to formatively assess their students. Teachers encouraged students to express their ideas explicitly in order to conclude how they understand the topics they have learned. Others asked students to prepare portfolio protocols using shared files, such as google docs. Some of the teachers asked their students to summarise the content they had learned and to present it using PowerPoint presentations.

INNOVATIVE LEARNING SYSTEMS THAT ARE INSPIRED BY THE PANDEMIC

So far, I have briefly reviewed some of the challenges teachers faced during the Covid-19 period, and I reviewed how teachers used digital tools to deal with these challenges. My main argument in this talk is that the challenges that manifested during the Covid-19 period and the ways teachers utilized digital tools to deal with these challenges created unique opportunities for researchers and software designers to develop innovative digital tools, which can help in overcoming some of the challenges and can improve instructional practices after the pandemic. To support my argument, I will discuss three examples of digital tools designed to help teachers deal with these challenges. The first system is called SAGLET: It is a system aimed at fostering inquiry-based Learning through social interaction. The second system in the MathematicX. This system is designed to help teachers in organizing the learning process. It allows teachers to choose a topic they want to teach and to set a generic question. The system will automatically create content similar to the topic that has been chosen. The system will present the content step by step to the students to help them progress at their rate. The third system is Dash4Emotion, which alerts teachers on the students' sentiments in real-time.

SAGLET

SAGLET (Segal et al., 2017) is designed to support collaborative learning and inquiry-based learning using educational software in classrooms. SAGLET (Figure 1) augments existing online learning environments to include technology tools that are capable of (1) recognizing critical moments of emergent learning in groups that interact with one another using educational software and (2) visualizing salient information to teachers. SAGLET provides a set of alerts that the teacher may use in order to handle the orchestration of multiple groups (in up to 12 virtual rooms) engaged concurrently in a learning task. Although the provision of critical moments seems clearly useful, caution is required in using alerts in learning processes.

Segal et al., (2017) integrated SAGLET with VMT software. VMT includes a Geogebra applet shared by all participants so as to provide opportunity for collaboration on geometrical tasks (Stahl, 2013). Figure 1 displays the VMT interface, which is comprised of two parts. First, it includes an inquiry space where small groups of students can share their mathematical explorations and co-construct geometric figures online (part A). When one of the participants drags or constructs a geometrical figure, all the others can see the changes in the figure. As shown in Figure 1, VMT also provides a chat window (part B) in which students can write their ideas and share them with their peers.

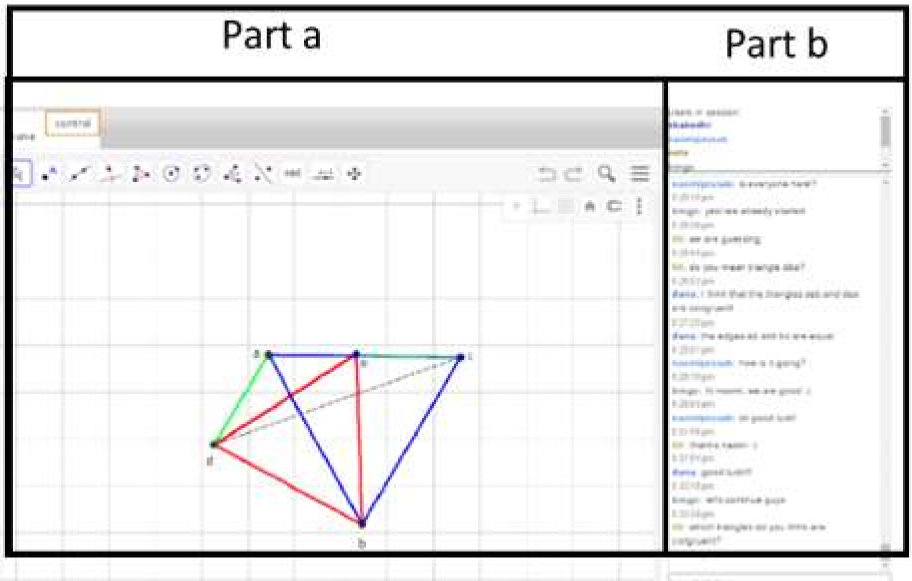

Figure 1: The VMT interface.

SAGLET allows teachers to observe, online, the work of groups of students engaged in learning tasks with VMT in different virtual rooms and to intervene whenever they think it appropriate. As the learners progress in their group work, SAGLET informs the teacher about critical moments through alerts. Figure 2 shows an instance of windows observable by the teacher. In this case, the teacher was informed about an off-topic (blue frame) discussion in virtual room 454 and a technical problem (yellow frame) in virtual room 486. The alerts are easily visible as a colored frame appears according to the alert type. Teachers can ignore alerts or decide to respond by entering any virtual room they wish.

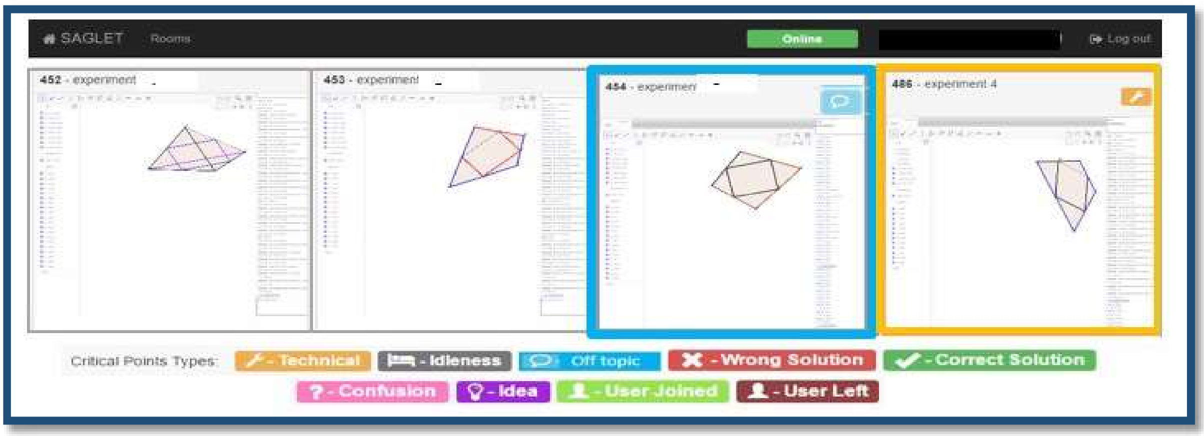

Figure 2: The teacher is informed about an off-topic discourse in room 454 and about a technical problem in room 486.

MathamticX

The MathematicX platform (Lisutta, et al. 2023), which is still under development, has two main interfaces. The first interface is for students, and the second is for teachers. In this platform, the teacher constructs the content to be learned. The system assists the teacher in constructing the content and collects data about the students' learning processes. The

MathematicX system provides a list of topics from which teachers can choose. For example, let us assume the teacher wants to teach the students how the 'm' and 'n' parameters affect the linear function graph. In this case, the teacher should choose the topic "linear function." The MathematicX system provides the teacher with several types of functions, such as polynomial, trigonometric, exponential, etc. (Figure 3a). The teacher then chooses a polynomial and determines the degree of the function. In our example case, the teacher chooses the degree of the function as 1. In the next step, the teacher chooses the request that may guide his/her students to learn the intended topic, in our case, the effect of the 'm' and 'n' parameters on the linear function graph. The teacher can choose the request from a set of ready-made requests (Figure 3b). Once the function type and the item's request are determined, the teacher must set the variation domain of the parameters (i.e., the 'm' and 'n' parameters) (Figure 3c).

This design principle was motivated by the variation theory of learning (Marton et al., 2004). Variation theory defines learning as a change in the way an object of learning is discerned: how it is seen, experienced, understood. According to variation theory, an object of learning can be formulated in three different ways of increasing precision, in terms of: content (e.g., a linear function), educational objectives (e.g., generalizing family of linear function), and critical aspects, which the learners should simultaneously discern to make the object of learning their own.

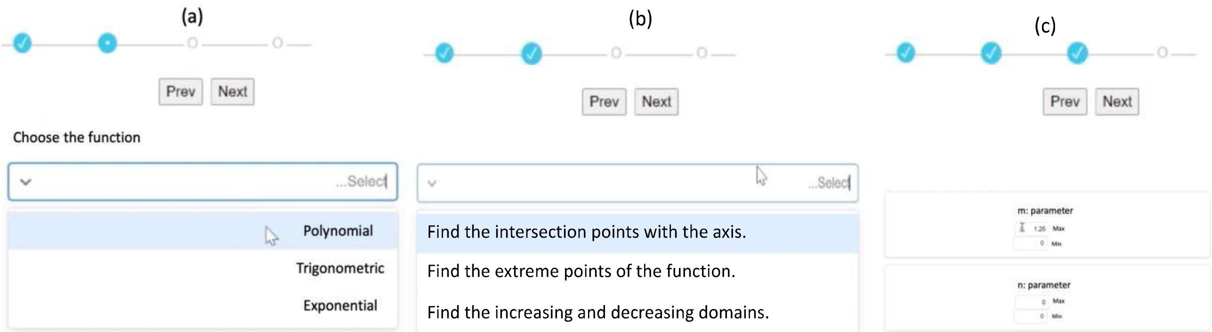

Figure 3: (a) type of function that can be chosen, (b) ready-made request for the items, (c) the interface of setting the value of the parameters.

According to Marton et al. (2004), learning occurs through the separation of the object of learning from other objects from the same dimension of variation, which refers to the aspect to focus on (e.g., varying 'm' while keeping 'n' constant). After discerning the meaning of the variation of one parameter, the whole must be put together again to simultaneously experience certain aspects of the object of learning: this pattern is called fusion. To endow an object of learning with meaning, it should be constrained with other objects (e.g., to identify a function as linear, it should be constrained with functions that are not linear). Once the meaning is found through the contrast pattern, the generalization of the object of learning becomes necessary.

Inspired by the variation theory, the MathematicX system allows the teacher to separate parameters by determining which parameter will vary. It also allows the teacher to constrain one type of function with another, and it allows the teacher to fuse the variation of several parameters at the same time.

Once the teacher has prepared the learning sessions, the system will automatically generate items that meet the principles he/she has set. To move from one learning session to the second, the teacher should decide the number of the requested successful attempts the students are required to make. For example, the teacher can decide that after three successful attempts of the same type of item, the system will move to the second type of item. If the students are not able to solve correctly, the system can give them hints on how to progress. The system also allows the students to chat and send messages to the teacher and the students.

The teacher interface in the MathematicX interface includes a dashboard that contains descriptive statistical analysis per student and class. The system provides, for example, data concerning how many items each student has solved, the distribution of the correct and incorrect attempts, the time the students took to complete the session, and which students completed the entire session, and which did not.

Dash4Emotion

Dash4Emotion platform (Jbareen et al., 2023), which is also still under construction, has a different perspective than SAGLET and MathematicX. Dash4Emotion is an innovative system for monitoring students' emotions (Figure 4) and alerting teachers about their students' emotions in real-time. It is designed to help teachers be aware of the students' emotions while they are learning mathematics in order to allow the teacher to regulate the students' emotions if needed. The Dash4Emotion system uses *AI emotion recognition,* a field of computer vision focusing on facial *emotion detection* and automatic *sentiment* analysis from visual data. Facial *emotion detection* software is a tool that makes it possible to detect and determine displayed human *emotions*.

Figure 4: Set of sentiments that the Dash4Emotion can detect.

To avoid an overload of alerts to be sent to the teacher, the designers decided to inform the teacher about the students' sadness, fear, anger, only if the students maintained this sentiment for more than 10 seconds. Of the surprised sentiment, the system immediately informs the teacher. Of happiness, the teacher is informed only after 60 seconds, and is not informed at all when students are neutral or disgusted.

FINAL REMARKS

My aim in discussing the three systems is to illustrate how the challenges manifested during the Covid-19 period helped in reflecting on developing or looking for digital systems that might overcome some of those challenges. I am not arguing that the learning systems discussed in this paper are deemed to solve all the challenges created during the Covid-19 period. Nor that the systems we are developing is the best solution for overcoming the challenges.

The SAGLET system, for example, which provides a shared space for inquiry and allows grouping students to foster collaborative learning, can follow the students' learning and inform the teachers on how the students are progressing or even being hindered. SAGLET, by allowing the teachers to be aware of what is going on in the virtual rooms, can help in overcoming challenges related to applying the collaborative learning and contemporary instruction approach, which arose as a result of the remote learning during the COVID-19 period. In addition, the data provided by SAGLET and sent to the teacher can support the teachers in formatively assessing their students. These data can elicit evidence of students' understanding (Cusi et al., 2022). The MathematicX platform, by allowing teachers to create the content they want to teach and by allowing each student to progress at their own rate, can be a solution for supporting students who face difficulties. The Dash4Emotion platform can be considered as a solution for detecting students' emotions, which was one of the main challenges that teachers faced during Covid-19 period.

Although, I hope, the Covid-19 lockdown period is over, it has presented opportunities for raising new ideas on how to design innovative learning environments. I believe that the learning platforms that have recently emerged can influence the educational system in the future. These systems can be used not only during the pandemic period but also during ordinary days. I should note that the three systems briefly presented here are not the only ones available today, nor are they the best. Other systems, designed to overcome other challenges, are also available today (e.g., Asymptote) (Barlovits et al., 2022). This effort, without any doubt, is very blessed. The emergence of such systems requires systematic research to understand how teachers and students use such systems, and how they are able to help them overcome the challenges they meet. As we learned from our research on the SAGLET system, this learning platform solves some issues, but its use in the classrooms also creates new challenges (Schwarz et al., 2021). International collaboration is needed to study how the use of such digital tools may help in overcoming the challenges and what other kinds of challenges the new technologies may create.

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DIGITAL LEARNING GRAPHS WITH ASYMPTOTE – STUDENTS FEEDBACK

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Abstract. The success of digital technology in mathematics education includes the design of digital tools. The ASYMPTOTE system has been developed and improved since March 2021 an effective, powerful, and user-friendly digital tool for online mathematics education. This article aims to present an exploratory study of the use of the ASYMPTOTE system in the classroom. The participants of this exploratory study are engineering students of different degrees of the School of Engineering of the Polytechnic of Porto (ISEP) and in different subjects. In some subjects, the ASYMPTOTE was used in an evaluation context and in others in a training context.

Key words: ASYMPTOTE, digital tools, mobile learning.

INTRODUCTION

The pandemic, caused by COVID-19, affected the lives of millions of people and the closure of teaching activities, forced universities and polytechnics to reorganize and move from face-to-face teaching to online teaching. However, this reorganization was not thought about calmly, it was motivated by the need of the moment, which is why this teaching was called emergency remote teaching because it is a solution for the moment of confinement (Viamonte & Pinto, 2022; Barlovits et al., 2022). However, this teaching resulted in many gaps in the students' learning process and one of the most principal issues that appeared during the confinement was the guarantee of equal opportunities for access to digital education. Netta Iivari and colleagues address the digital transformation started in education due to the COVID-19 pandemic (Iivari et al., 2020). They mention that some problems with access to technology meant that not all students were in an equal position to take part in digital education. The United Nations (United Nations, 2020) recommends as a top priority the right to higher education for all, within a framework of equal opportunities and non-discrimination.

Technology is fully embedded in society, being part of people's lives in many ways and being increasingly implemented in our daily lives. Students and teachers must adapt to this evolution and, therefore, it is necessary to implement new methodologies, with the use of technological tools such as mobile phones and the internet, which are directed to the needs of students (Esteve-González et al., 2015; Haleem et al., 2022). The idea of adding digital resources such as the use of specific applications in the classroom as an auxiliary teaching method brings flexibility for both students and teachers. In this way, using the students' own mobile phones, it is possible to help in the assimilation of knowledge, and information and to facilitate learning, since the student can study at any place and time, being able to provide the teaching-learning process becomes more pleasant and stimulant. When the mobile phone is used with an adequate methodology in the educational environment, it is possible

Caldeira, A., Figueiredo, I., Gavina, A., Pinto, I., & Viamonte, A. J. (2023). Digital Learning Graphs with ASYMPTOTE – Students Feedback. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 145–152). WTM. https://doi.org/10.37626/GA9783959872522.0.17

for the student to see that it can be used for purposes that bring benefits to their knowledge and that this is also a means of communication, which is essential in the teaching-learning process and makes this process simpler and easier. However, the use of these technological tools must be directed in an adequate and well-defined way, to promote inclusion efficiently (Iivari et al., 2020; Mehdi et al.., 2020).

Since March 2021, the ASYMPTOTE system has been developed and improved. ASYMPTOTE stands for Adaptive Synchronous Mathematics Learning Paths for Online Teaching in Europe (https://www.asymptote-system.eu/en/portal-en/#!/), and the project aims to develop an effective, powerful and user-friendly digital tool for online mathematics education. This system consists of two components, a web portal for teachers and a smartphone application for students (Barlovits et al., 2022).

In this article we will present an exploratory study of the use of the ASYMPTOTE system in the classroom. The participants of this exploratory study are engineering students of different degrees of the School of Engineering of the Polytechnic of Porto (ISEP) and in different subjects. After this introduction, we will describe the ASYMPTOTE system. We begin by presenting and describing the system and, then we will describe the scenario in which this exploratory study was carried out: the several courses, the different subjects and, if the ASYMPTOTE was used in class as a training or as an evaluation tool. Next, we will investigate the impact of the ASYMPTOTE on students, an anonymous survey was conducted to assess the students' opinions about this experience. The goal is to present the results obtained in the survey and we end with some considerations and conclusions.

APPROACH

The integration of digital technology into the teaching and learning process has been fundamental, resulting in significant changes in education. The use of digital technologies gives teachers the opportunity to design engaging learning opportunities in the courses they teach. In this article we will use the ASYMPTOTE system in the classroom in a school of engineering and in different subjects.

The tool

The aim of ASYMPTOTE project is to develop a system for synchronous and adaptive online learning. The ASYMPTOTE system has two components: a web portal and an App.

- A web portal where teachers can create tasks and learning graphs, or even, search and choose tasks and learning graphs that are available in the system. Teachers can cluster some tasks in a sequence with multiple levels of difficulty, for a specific or general topic of mathematics.
- An App for smartphones where students can complete assignments and browse the learning graphs. In the App it is possible to visualize the tasks and the learning graphs. Students, strolling the learning graph, solve the tasks and get synchronous and systematic feedback. The applications run on Android and iOS mobile devices.

The web portal also provides teachers the Digital Classroom, where, the teacher can monitor their students' work in real time. A chat is also implemented, is for one-to-one synchronous communication between instructor and student (Larmann et al., 2021).

Tasks and learning graphs

Tasks in ASYMPTOTE are divided into three broad categories regarding their focus:

- training - includes mathematics and/or mathematical techniques;
- reasoning - requires the use a mathematical argument, interpretation, or explanation;
- modelling - real world modelling and problem solving.

The tasks must include some basic information, such as, the title and description and the author's information; core components, such as, the answer types, task type, sample solution, some hints, the assigned grade and tags. The answer type can be: exact value, interval, multiple choice, fill in the blanks, vector (exact value), vector (interval), set, fractions and matrices.

A learning path is defined as a sequence of tasks which are designated to assist the student in improving their knowledge or skill in a particular subject area (Brusilovsky, 1992). In ASYMPTOTE, task sequences are presented in the form of a learning graph, which is defined in (Barlovits et al., 2022) as follows:

A learning graph is defined as a directed graph, where each vertex represents a learning activity (or task), based on a learning trajectory as the intended and expected way of learning.

The structure of ASYMPTOTE learning graphs ensures that the entire learning process is self-guided and autonomous and, it also aims gamification, where students receive points for a successful task-solving process, motivating and stimulating them for learning (Lieberoth, 2015). In the ASYMPTOTE, the learning graph consists in a sequence of tasks (see Figure 1):

- the main tasks (yellow), they are mandatory tasks and they in the center;
- support tasks (green), they are easier related tasks that can help to solve the main task afterward and they are on the right side;
- the challenge tasks (purple), with higher difficulty, challenging those students who want to dive even deeper into the topic, and they are on the left side.

As the tasks are carried out, those that were answered correctly will be marked. Students have four attempts per task, if they make a mistake on the first attempt, they don't lose points, and when they make a mistake, they are motivated to use the hints that are available in the task. When students respond to the task, they immediately receive feedback and a task resolution suggestion. Feedback, combined with effective instruction in the teaching and learning process, it can be very powerful in enhancing learning (Hattie & Timperley, 2007).

The learning graph in Figure 1 is named "Gauss Elimination Method and Applications" and can be downloaded in the ASYMPTOTE App by entering the corresponding code.

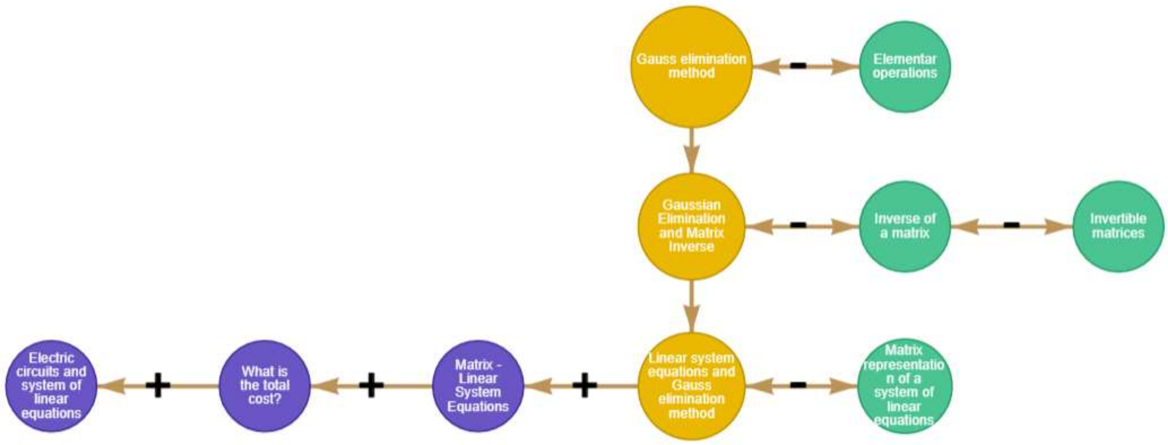


Figure 1: Example of a learning graph (CODE: g17203).

The setting

Our scenario is an engineering school, ISEP, and the participants involved were mathematics professors and engineering students. They are students of five different types of engineering degrees:

1 Biomedical Engineering (LEBIOM);
2 Civil Engineering (LEC);
3 Electrical Engineering – Power Systems (LEESEE);
4 Systems Engineering (LES);
5 Telecommunications and Informatics Engineering (LETI).

and three different subjects of the scientific area of Mathematics:

1 Calculus I;
2 Calculus II;
3 Linear Algebra.

The students of Biomedical Engineering attend the 2nd degree year and the rest attend the 1st degree year, totaling 409 possible students for the exploratory study.

It was proposed to the students of these engineering degrees courses that they solve a learning graph in class, in the context of an evaluation or in the context of preparation for an evaluation test.

Table 1 shows the relationship between the various engineering degrees, the subjects, the number of students enrolled in the discipline and the codes of the proposed learning graphs. It should be noted that not all students enrolled in the subjects took the proposed learning graph. There are students who, despite being enrolled in the discipline, do not attend classes for unknown reasons.

Engineering degrees	Subject	Number of students	Purpose	Codes
LEBIOM	Calculus II	61	training	G01337
LEC	Linear Algebra	119	training	G19364
LEESEE	Linear Algebra	91	evaluation	G49502
LES	Calculus I	71	training	G07299
LETI	Linear Algebra	67	evaluation	G69513

Table 1: Engineering degree - Subject - Number of students – Purpose and LG Codes.

RESULTS AND DISCUSSION

A questionnaire to understand the App's applicability as a tool for learning, training and assessment was given to students. The students' questionnaire contained eight questions of mixed nature, closed-ended and one question open-ended. In the closed-ended questions it was intended to evaluate the use and applicability of the App and the last one is an open-ended question where the students were asked to indicate the weaknesses and the strengths of the App. Of the student population (409) enrolled in courses that used the App, 176 participated in the questionnaire.

Two variables were defined to characterize the participants in this survey by questionnaire: gender and number of enrollments in the course. Figure 2(a) reveals that male participants were remarkably higher than females and Figure 2(b) shows that most students are attending the course for the 1st time.

(a) (b)

Figure 2: (a) Participation according to gender; (b) Number of attends in the course.

Regarding the use of the application, the two questions were made to students:

- Should the ASYMPTOTE App be a tool to use in learning and training during theorical an practical classis?
- Should the ASYMPTOTE App be a tool for evaluation?

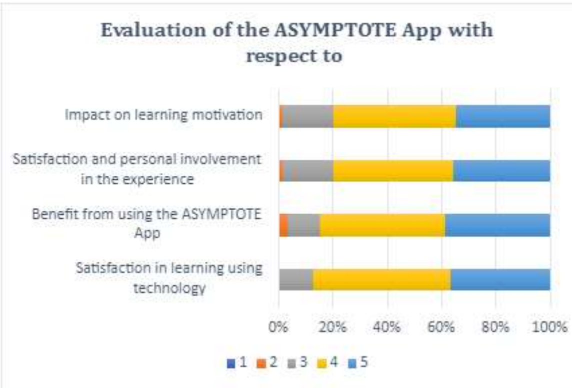
About the use of the App as a tool to be used in classes, most of the students (79%) agree with its use, but different answers were given about the use of the App as a tool for evaluation. Only half of the students (54%) agree with its use for evaluation. Note that 25% disagree with its use. Table 2 shows the results.

	Should the ASYMPTOTE App be a tool to use in learning and training during theorical an practical classis?	Should the ASYMPTOTE App be a tool for evaluation?
Strongly disagree	1%	8%
Disagree	5%	17%
Neither agree nor disagree	15%	21%
Agree	46%	33%
Strongly agree	33%	21%

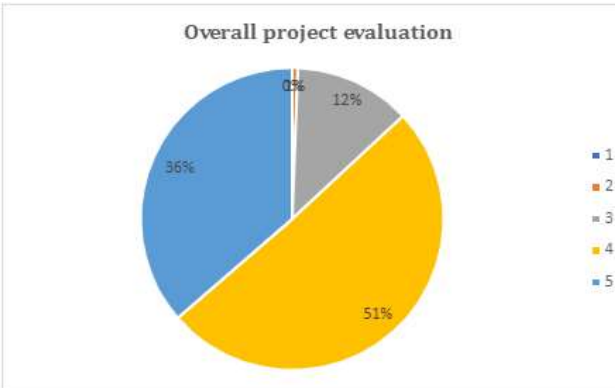
Table 2: Use of ASYMPTOTE App application.

The students were asked to evaluate the ASYMPTOTE App with respect to five items: impact on learning motivation, satisfaction and personal involvement in this experience, benefit from using the App, satisfaction in learning using technology and overall project evaluation. A Likert-type scale on satisfaction was used to specify students' levels of satisfaction, where 1 means that students are very dissatisfied with ASYMPTOTE App and 5 means that they are very satisfied. As can be seen in Figure 3(a), the "overall project evaluation" is very good. The fact that a student can study (or be assessed) using an app, which allows suggestions, immediate feedback and access to task resolution, makes the project very appealing.

In the first four items a great percentage of students (more than 80%) are somewhat/very satisfied with the App, only a residual percentage (less than 3%) states that they are somewhat/very dissatisfied. Most students (87%) are overall satisfied with the App, see Figure 4(b).



(a)



(b)

Figure 3: (a) Evaluation of ASYMPTOTE App; (b) Overall project evaluation.

Weaknesses and the strengths

Students were asked to justify their answer by giving their opinion about the project relative to weaknesses and strengths. Some students considered that the project was very interesting, and it was an opportunity to gain new skills. Others point out existing bugs on App and internet network failures as points to improve. Some of the answers are expressed in Table 3.

Weaknesses	**Strengths**
"Existing bugs"	"Having the resolutions and the large amount of exercises"
"Some internet failures can harm the student"	"Using technology to learn revealed greater interest on my part"
"For a global assessment of the subject, I don't think it's a fair way of evaluating the discipline, mainly because I personally think that mathematics has to be expressed on paper, with calculations"	"I think the implementation of ASYMPTOTE was a very creative idea, which in a way captivated the students' interest as it was a different platform than usual. The level system transforms learning into a fun and more relaxed challenge, the possibility of having "clues" in the exercises helps to clarify possible doubts in solving them and the demonstration of the exercise solution and its calculations at the end make these activities like a "mini-lesson"".
"Too bad it doesn't have computer support"	"The App is easy to use and well organized, in terms of learning it is a good teaching tool"

Table 3: Students' opinions about the App.

CONCLUSIONS

The present study is based on the student's responses to a survey about the use of ASYMPTOTE in the classroom and intends to assess the advantage for students' motivation and learning. The survey had several questions on a 5-point Likert scale and at the end had an open-ended question where students were asked to point out the main negative and positive points of their experience with ASYMPTOTE. Although no conclusions can be drawn as there was no control group, based on the data it seems that students believe that using ASYMPTOTE is efficient for their learning and motivation.

Some responses suggested that the use of ASYMPTOTE as a teaching tool brings several benefits to the educational process, such as increasing individual participation in the teaching-learning process and the possibility for students to manage and plan their own learning process. As can be deduced from the data, in terms of ease of use and usefulness, the average score of participants is above average. These dimensions illustrate the benefits of using ASYMPTOTE and these results are also in line with other research (Gan & Balakrishnan, 2014; Mehdi et al., 2020), claiming that the adoption of mobile technology in learning can improve teacher-student interaction and factors such as ease of use, self-efficacy, and engagement an important role in the uptake of mobile digital learning.

The results show that, in general, the App was well accepted by the students. Most of them agree that the App should be used as a tool to be used in classes and most of them are overall satisfied with the App. While mobile learning can never fully replace traditional learning, it appears that it can increase motivation and contribute to more effective student learning. Mobile phones have come a long way from being seen as distractions in the classroom to now being seen as tools that can help students. There are several benefits to using mobile phones as a teaching aid, including better learning outcomes, greater student engagement, and an easier ability to keep students up to date on assignments. However, as with all tools, teachers need to plan to ensure these devices are used properly in the classroom.

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A CASE STUDY FOR THE PEDAGOGICAL EVALUATION OF THE ASYMPTOTE SYSTEM

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Abstract. *This paper examines how primary students use the ASYMPTOTE system and whether it helps them overcome their misconceptions about fractions. Therefore, a case study was conducted for 6th grade students (Primary School) during their regular school schedule. The results proved that some of the students who engaged systematically with ASYMPTOTE's affordances managed to overcome, to a considerable extent, their misconceptions with the support of their teacher.*

Key words: ASYMPTOTE, mathematics teaching, primary education.

INTRODUCTION

The challenges that the covid-19 pandemic brought to the educational system (Barlovits et al., 2021; Mishra et al., 2020; Zhang et al., 2020) arose the need for the development of new learning environments that can guide the learner remotely (Flores & Swennen, 2020; Hall et al., 2020), particularly in Mathematics education, where the integration of technological means in the teaching process had been more conservative in previous years (Chronaki, & Matos, 2013). In this research, the ASYMPTOTE system (www.asymptote-project.eu), which is a modern tool for teaching and learning Mathematics remotely from primary education to university level, was studied to identify its pedagogical validation. The ASYMPTOTE system's development started on 2021 as part of the ASYMPTOTE project (Adaptive Synchronous Mathematics Learning Paths for Online Teaching in Europe), funded by ERASMUS+ KA2, with the participation of seven institutions from five European countries (Germany, Greece, Italy, Portugal, and Spain).

THE ASYMPTOTE SYSTEM

The ASYMPTOTE system consists of two components, a web portal and a mobile app (Barlovits, et al., 2022). In the web portal teachers can create their own tasks or use already existing ones from a rich task repository. The main goal for the teacher is to design organized collections of tasks, i.e., a *Learning Graph* (LG) with multiple levels of difficulty, for a specific topic of Mathematics. The web portal also contains a Digital Classroom functionality (DC) that provides the opportunity of direct communication between the teacher and the students and personalized support (Barlovits, et al., 2022). In parallel, through the app the students can browse and interact with a specific LG that the teacher assigned them on, having systematic, synchronous feedback on their entered solutions (Barlovits, et al., 2022). The mobile application is established upon gameplay elements such as a reward-based system that increases students' engagement (gamification). In this research emphasis is given on the use of the system's affordances, as described in Barlovits, et al. (2022), by the students.

Fesakis, G., Triantafyllou, S., Tzioufas, N., Koutsomanoli-Filippaki, D., & Volika, S. (2023). A Case Study for the Pedagogical Evaluation of the ASYMPTOTE System. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 153–160). WTM. https://doi.org/10.37626/GA9783959872522.0.18

RATIONAL AND RESEARCH QUESTIONS

The mathematics topic that was selected for this case study is fractions, mainly because it is an important part of Mathematics (Bailey et al., 2012) and a conceptual barrier for students (Gabriel et al., 2013; McMullen et al., 2015), relating to a series of common difficulties and misconceptions in Primary Education. Adopting the findings of Aliustaoğlu et al. (2018), we examined the following areas regarding fractions, in which specific misconceptions for students of 6th grade have been identified: part-whole relations, representation on the number line, comparison and operations (addition).

The aim of this research is to examine how students will use the affordances of the ASYMPTOTE system to overcome their misconceptions. In parallel, this research glances at the teacher's/researcher's difficulties on the use of the DC tool. Thus, the following research questions should be answered through this study:

(1) Which of the affordances of ASYMPTOTE do students use the most and how?
(2) To what extent did the affordances that the students used, helped them overcome their misconceptions about fractions?
(3) Which of these affordances were most useful according to the students' opinion?

METHODOLOGY

For this research, an exploratory case study (Yin, 2009) was developed using a group of students as the unit of analysis in order to study ASYMPTOTE in real classroom conditions. For the data collection, the research followed a mixed quantitative and qualitative approach. More specifically, a Pre-test and a Post-test questionnaire were used to evaluate and estimate the impact of the teaching intervention with ASYMPTOTE and a specifically designed LG on the students' learning of fractions. Also, personal interviews were conducted in combination with a Likert-scale questionnaire for assessment of the usefulness of ASYMPTOTE's affordances. The questionnaire is available here: shorturl.at/ePQ37. Finally, the video recordings of the interaction of students to the system were analyzed.

Research conditions & process

Firstly, for the design of the Pre-test and Post-test questionnaires, the tasks of the data collection tool used in the research of Aliustaoğlu et al. (2018) were implemented with minor adjustments. The test that was used in this study can be found here: shorturl.at/kwxDU.

For the teaching intervention, two LGs were designed in ASYMPTOTE's webportal, in order to support the students' learning, regarding their misconceptions in the specific areas of the fractions mentioned above. The first LG "Fractions 2" (https://www.asymptote-project.eu/de/portal-de/#!/graph/g38312) covers the part-whole relation, the representation of fractions on the number line and the comparison of fractions. The tasks included in it were adapted to the corresponding questions per section of the Pre & Post test. Since there was no previous introductory teaching in fractions, special emphasis was given to the utilization of *Support tasks*, *Hints* and *Sample Solutions*. The first LG was structured in four series of tasks (Figure 1, left) with the criterion of supporting one subject to another in a cognitive continuum. In the beginning of the LG there was an introductory *Main task* on the parts of the fraction followed by a *Main* and a *Support task* regarding the thematic "part-whole". Three tasks -*Main, Support* & *Challenge*- were also created for the comparison of fractions, focusing on the relationship between the numerator and the denominator. To help students comprehend how the number line is divided into parts in order to place the fraction on it, one *Challenge*, one *Main*, and three *Support* tasks were created for the representation of a fraction on the number line, which was thought to be the most demanding topic.

The second LG was created to teach the topic of addition in fractions. The LG "Fractions 3.2"(https://www.asymptote-project.eu/en/portal-en/#!/graph/g35315) was structured (Figure 1, right) with the same logic of the cognitive continuity of the subjects which in this case were *Equivalent fractions, Comparison* and *Addition of fractions*. The *Comparison of Fractions* and the whole LG in general are focusing on the strategy of creating like fractions.

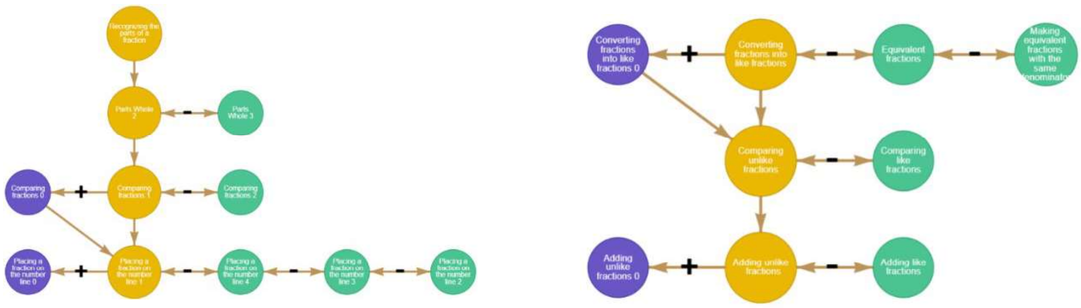

Figure 1: LG Fractions 2 (G38312) and LG Fractions 3.2 (G35315) (from left to right).

After the design of the Pre-test, Post-test, and the development of the two LGs mentioned above, five teaching hours were devoted to familiarizing the students with the ASYMPTOTE system, so that they could use all of ASYMPTOTE's affordances before the start of the teaching intervention. During this stage, three LGs were used (LGs codes: g47109, g05316 and g12319).

Participants and data collection

The case study was carried out face to face in the 6th grade of a Primary School in a suburban area of Rhodes, Greece, in a classroom with 7 students (1 boy, 6 girls). Firstly, the Pre-test was given, and the misconceptions of the students were determined. The next few days, two two-hour sessions with the LG application took place in the classroom, where the teacher/researcher was communicating with the students only through the ASYMPTOTE DC chat. The screen of both the students' devices (tablets and mobile phones) and the teacher's laptop were recorded. Detailed observation of the chat-log and the screen recordings was documented for each student to locate incidents of learning, support provision and the utilization of ASYMPTOTE's features in general. After the teaching intervention, the Post-test was filled by the students. On the same day, through personal interviews, the Likert-scale questionnaire for assessment of the usefulness of ASYMPTOTE affordances was completed.

FINDINGS

Patterns of use

The data collected from these two sessions led to the detection of two main patterns of student use of the system: a) *Using ASYMPTOTE as a learning tool* and b) *Using ASYMPTOTE as a quiz app*.

In the first category that is constituted by the majority of the students, the system is clearly used as a learning tool and the students' patterns of use exhibit signs of a learning orientation. Moreover, in this category we can distinguish two subcategories regarding the use of the learning supporting functions: a) *using Hints as the main helping tool* and b) r*esorting to asking the teacher for help through the Chat*. Students with pseudonyms ES, EA and EX, for example, followed a strategy of carefully reading the content of the tasks and prioritizing the use of *Hints*, either from the beginning, as additional information or in the event of a mistake in order to redefine their solution path. They generally acted autonomously and communicated with the teacher only in cases where the help from the *Hints* was not sufficient. For example, student EA, having completed five tasks, contacted the teacher for the first time after submitting two wrong answers, because she couldn't understand why they were wrong. In the second subcategory we have students who rely more on the help of the teacher through the *Chat* features. The students in this category seem more cautious and insecure and to avoid making mistakes, they contacted the teacher through written or voice recorded messages. EM for example, wasted valuable time on a task waiting for the teacher's response to a procedural issue that she ultimately solved on her own. However, both EM and EB subsequently started using more *Hints*, alongside communicating with the teacher.

On the contrary, in the second category, students do not show much interest in the learning part of the session but focus on using ASYMPTOTE as if it was a common knowledge quiz. For example, students EK and EP rushed through the LG without noticing the details of the tasks and chose their answers almost at random. An indication of this pointless browsing of the LG was that they didn't use the affordances of the application that aim to support the learning process, such as *Sample Solution, Hints* and the *Chat*. EP, for example, did not use any *Hint* and didn't communicate with the teacher, except at the end to declare that she was finished.

The Sample Solution

During "Session 1" of the DC almost no students paid the expected attention to the *Sample Solution* after correct or incorrect answers. An exception was student EB who seemed to carefully read the *Sample Solution* in most of the tasks as opposed to EC who hardly ever opened them. In "Session 2" however, a small increase in the use of Sample Solutions was observed.

The Give up button

During "Session 1" there were only two skipped tasks and 1 Not completed, while in "Session 2" there were 6 Skipped tasks and 10 tasks Not completed. The majority of the Not completed tasks were *Challenge tasks* (1 in "Session 1" and 8 in "Session 2"). This is due to the fact that "Session 2" was perceived as a more difficult session and because, after "Session 1", the students realized that the *Challenge tasks* were optional. In addition, we could tentatively say that the gamification function of the application didn't have the expected effect on the students of this research, who did not seem to care whether they would lose points by not completing the *Challenge tasks*.

Learning incidents

Worth noting learning incidents were identified during the analysis of the users' activity (screen recording & event logs of the system) and in some cases they indicate a shift in students' initial misconceptions. More specifically on the group of tasks included in LG Fractions 2, EA, who initially completed the first two tasks (recognizing the parts of the fraction, Parts Whole 3) without opening Hints, in the third task (Parts Whole 2) she started with an incorrect answer which forced her to open Hint 1. This led her to choose the correct answer but before clicking on the "check" button she opened and studied Hint 2, to make sure of her answer. This specific incident of learning through *Hints* was also reflected positively in EA's performance in the Post-test, where she corrected the relevant question 1c and by extension her initial misunderstanding (Figure 2). The *Hints* feature of ASYMPTOTE seems to be able to play a role of formative assessment in online learning of mathematics.

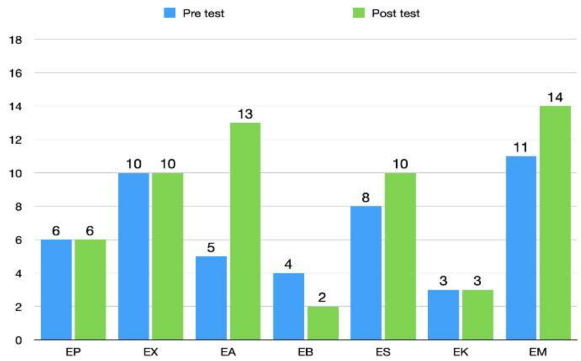

Figure 2: Pre & Post test results of each student.

Another interesting incident of learning was recorded in the task "Comparing fractions 1", where student EA gave the same wrong answer twice and as a next step, she texted the teacher for help. This dialogue followed:

EA: Mr, it tells me it's wrong but it's right, why?

TEACHER: Where, in what exercise? Tell me exactly.

EA: The third yellow.

TEACHER: You will put a letter as the answer (A or B or C or D). Pay attention, it says from the smallest to the largest.

The help from the teacher seems to have influenced the reasoning of EA, who immediately gave the correct answer. In this case EA did not use *Hints*, changing her usual practice probably because she was sure of her answer.

In "Session 2" there were noted two incidents of learning that led to an improvement in the performance of two students in the Post-test. In the first, ES, following his favorite Hints-focused usage pattern, opened the *Main task* "Adding like fractions" and gave three wrong answers. Between them he opened and studied the *Hints* again and again and then proceeded to the *Support task* "Adding like fractions". There he easily gave the correct answer and studied the *Hints* again to go back to the previous task where he opened Hint 3 once more. It seems that he was not getting sufficient help, so he gave another wrong answer. It is noteworthy that ES was intently searching for the information that would help him understand how to solve the *Main task* in the *Hints*, then turned in the *Support task* and finally again in the *Hints*. The fact that he repeatedly studied the same *Hints* and opened some of them even after his final answer, further emphasizes this finding. Even though he failed to solve the "Adding unlike fractions" task, the information he got from this search helped him give the correct answer to question 5 of the Post-test.

In a corresponding incident, EA started with two incorrect answers from the *Support task* "Adding like fractions". The *Hint* that she then studied helped her not only to give the correct answer but also to improve her Post-test performance in questions 4a and 4b. In the *Main task* she gave four wrong answers. Between these answers, EA, as ES did, opened and studied the *Hints* many times and communicated with the teacher. The interesting fact in this incident is that EA, after the last wrong answer, in addition to the three *Hints*, studied the *Sample solution* as well. Thus, despite the incorrect answer, the learning process that followed helped EA, similarly as ES, to improve her performance in task 5 of the Post-test.

Difficulties

The usage recordings showed that the teacher/researcher encountered some difficulties while using the system. An indicative example is when student EB asked for clarification about the task "Comparing fractions 2" and the teacher mistakenly thought that EB was in the task "Placing a fraction on a number line 3" and replied accordingly. This mix-up caused confusion to the student who, after finally receiving the correct clarifications from the teacher, spent a lot of time re-reading their dialogue before understanding the information provided to her and finally giving the correct answer.

This incident highlights the difficulty for the teacher in monitoring the usage activity of all participants in a session. It is a process that obviously requires enough practice and familiarity with the application for the teacher to be able to successfully check and respond immediately to students' messages, while cross-referencing the information in the "participants" menu and the detailed usage activity in the "events-log". Given the fundamental targeting of the ASYMPTOTE project in remote learning, this is a critical feature on which a large part of each session depends. The teacher should be able to intervene easily, when necessary, with general or personal messages, in order to help the students readjust their pattern of use and overcome difficulties. Indicative is the inability of the teacher in this study to check in time the events-log of the user EP, who rushed to complete the tasks without taking into account the sporadic text messages from the teacher, such as "Anyone who needs help or explanations send me a message to help" or "Don't rush".

The usability questionnaire

Some interesting results also emerge from the usability questionnaire analysis. As seen in Figure 3, which presents the sum of all students' choices, the features *Support tasks*, *Hints*, *LG map*, *Recorded messages*, *Interface*, and *Gamification* stand out higher in students' preferences, while the *Challenge tasks*, *Sample solution*, *Written messages*, *Picture messages*, and *Live snapshot*, got a lower grading by the students.

In more detail, it seems that the students in this study consider the *Support tasks* more useful than the *Main* and C*hallenge tasks*, as expected. Regarding the other help functions, *Hints* outperformed the four chat communication options (8a to 8d), but *Recorded messages* were the most popular teacher communication option. The students don't seem to consider the *Sample solution* as a very useful feature as it gained one of the lowest scores. EB and EA users, however, rated this feature with a 5 on the Likert scale which is consistent with the observations recorded from the usage activity analysis above. On the contrary, it is noteworthy that while no interest was observed in the *Gamification* function from the usage activity, the students' opinion of its usefulness was very positive. Finally, a particularly useful illustration, according to the users of the study, was the *map of the LG*, and the overall *Interface* of the application.

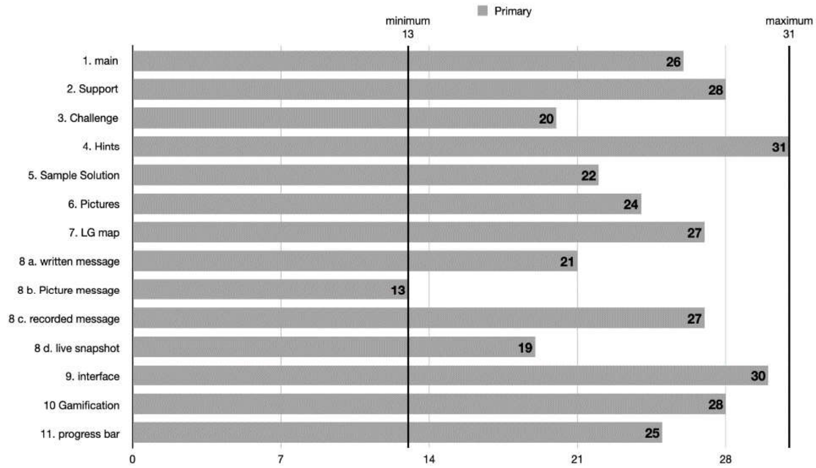

Figure 3: Results of the usability questionnaire.

CONCLUSION

In this research, a case study was developed to examine how students will use the affordances of the ASYMPTOTE system to overcome their misconceptions regarding fractions which is a challenging topic in mathematics education (Aliustaoğlu et al, 2018; Bailey et al., 2012).

As shown by the results of the usage analysis, *the students utilized more the following affordances of the ASYMPTOTE system*: the *Hints*, the *Chat* feature and the *Support tasks.* This finding also corresponds to the higher grade they gave to these features on the usability questionnaire. Regarding *the extent to which the system' affordances helped them overcome their misconceptions about fractions*, it was found from the students' Pre and Post test results that although there was not a significant improvement in overcoming their misconceptions, however, in the cases of students who did better on the Post-test, the LG design and ASYMPTOTE's affordances played a crucial role. Finally, *according to the students' opinions, the most useful affordances were* the *Support tasks*, the *Hints*, the *LG map*, the *Recorded messages*, the overall *Interface of the app* and the *Gamification* aspect of the system. Their preferences were in line with the way they used the app, with the only exception being the *Gamification* aspect, since in both sessions the students didn't pay attention to the grading system and especially in "Session 2" they didn't make an effort to solve the *Challenge tasks*.

To conclude, the data collected from this study indicate that students consider most of ASYMPTOTE's affordances useful. The DC can be used to support fruitful interactions among the teachers, the students and the LGs resulting in quality learning and misconception overcoming even in hard topics such as the fractions in primary education. Regarding the pedagogical validation of the ASYMPTOTE system, the presented case study provides supporting evidence that the system supports the key requirements for effective teaching and learning during Emergency Remote Teaching that are mentioned in Barlovits et al. (2022) such as Personal interaction and Formative assessment. However, it is important to note that the sample of this study is too small to generalize and, since the system is still under development, further studies need to be conducted.

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ENHANCING COMPUTER SCIENCE PROJECTS WITH REMOTE TEACHING ELEMENTS

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Abstract. The COVID-19 pandemic called for the ad-hoc implementation of many different types of remote teaching elements such as the use of particular methods and software. Entering the post-pandemic period, the acquired knowledge during this time of crisis can be used to enrich daily educational practice. In this paper, we introduce the *Digitechnikum*, a project in which students work on socially relevant computer science projects over the course of one school year. Based on the experiences made with various remote teaching elements during the pandemic, we discuss the use of these tools and methods for computer science projects in a school context in the post-pandemic era.

Key words: Computer science projects, Digitechnikum, remote teaching.

INTRODUCTION

Even before the COVID-19 pandemic, remote teaching concepts for everyday school life were developed and tested in many countries (Zeinz, 2019). Those concepts were attributed many hopes and chances, such as the large-scale availability of education (Sibirskaya et al., 2019), as well as the promotion of self-learning skills (Erhel & Jamet, 2013) and the quick creation of individual learning opportunities (Bondarenko et al., 2021). Then, with the COVID-19 pandemic, the largest conceivable test scenario for these concepts emerged from one day to the next - the abrupt and widespread home-schooling described as "emergency remote teaching" (Hodges et al., 2020). Teachers and students got particularly challenged by the sudden requirement of digital skills and technical prerequisites and, last but not least, by a drastic decline in motivation on part of the students (Barlovits et al., 2021). Nonetheless, the pandemic has also given a decisive boost to the digitization of the European educational sector (Cone et al., 2022). Now, in the post-pandemic period, we want to take a closer look at remote teaching elements used during the pandemic and their potential use at school in combination with computer science projects.

Computer science is a subject closely related to digital tools and elements. The experiences made during the pandemic can thus benefit this subject more than others. In this work, we want to take a closer look at long-term computer science projects at school during which students must also work at home, since the time to meet in person (e.g. as part of regular school lessons) is limited. Remote elements hold the potential to enhance this hybrid project setting, enabling students to productively collaborate from afar and merging their achieved work in in-person meetings, where understanding and coordination is crucial. In the context of computer science projects and in light of the recent events related to the pandemic, we take a closer look at the extracurricular computer science project *Digitechnikum*. This project had to suddenly be conducted in a pure remote format, like the majority of educational programs during the pandemic. In this article we want to outline which remote teaching elements used in the Digitechnikum have proven to be enriching to collaboratively work hybridly on a computer science project at school. This includes elements of professional software development (keyword "agile methods"), as well as different software solutions.

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The methods and software will be discussed based on our observations as well as evaluation data from the Digitechnikum in order to shed light on the following higher-level question:

How can remote teaching elements used during the COVID-19 pandemic prove worthwhile for computer science projects in the post-pandemic period?

THE DIGITECHNIKUM AND ITS REMOTE ELEMENTS

About the project

The Digitechnikum[1], a project originating in Frankfurt, Germany, aims at enriching the educational landscape by offering additional computer science learning opportunities on a local level (Wetzel & Ludwig, 2020). In this extracurricular program, selected students aged between 14 and 18 years, work in teams of four on a soft- or hardware project addressing a socially relevant problem in Frankfurt over the course of one school year. After an initial kick-off weekend where the students get to know each other, find teams and formulate their project ideas, students meet every two to four weeks in rooms at Goethe University Frankfurt to work on their project. These meetings resemble informal gatherings in a maker space, as students are provided with a space to work on their projects and all necessary materials such as computers, hardware components and 3D printers. The students are encouraged to solve problems by their own means but can always resort to the help of mentors with various technical backgrounds. The project is currently running in the fourth year. In the first three years, a total of 58 students participated working on 13 different projects. Software projects were predominantly designed in the form of a mobile app. An example from year 2 is an app for tracking one's own behavior and evaluating how sustainable it is based on what the team called "flower power score" taking into account different variables such as eating habits and the use of public transportation. In year 1, a team developed a 3D-printed box containing an Arduino equipped with two ultrasonic sensors that could be fixed to a bike. If a car comes too close posing a threat to the biker, the box gives out a warning in form of a blinking LED and a loud beeping sound (for more details on this project see Läufer et al., 2023).

Remote elements of the Digitechnikum

The Digitechnikum project, which started in October 2019, was meant to occur entirely live and in person apart from the phases in which student teams continue to work on their project tasks at home. However, the COVID-19 pandemic made this impossible from March 2020 on. Hence, the rest of year 1 was conducted in a remote manner, though with some difficulties, as no "remote plan B" had existed so far. Since it was unpredictable, when the pandemic situation would be over, we developed a concept for conducting the Digitechnikum remotely from year 2 on. Figure 1 gives an overview over the phases in which the Digitechnikum could be conducted in person and the remote phases between October 2019 and December 2022 (today). Apart from the kick-off weekend, year 2 was conducted completely in a remote manner. Year 3 could run longer in person, though winter also made it necessary to continue remotely. In spring, in-person meetings could be reinstated.

[1] The Digitechnikum (https://sptg.de/projekte/wissenschaft-und-technik/digitechnikum) is a collaborative project of Polytechnic Foundation Frankfurt and Goethe University Frankfurt, funded by Polytechnic Foundation Frankfurt.

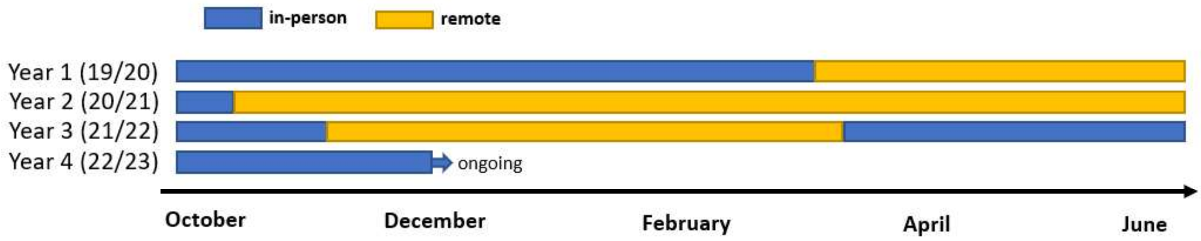

Figure 1: Overview of the phases in which the Digitechnikum could take place in-person and those in which it was conducted purely remotely.

The concept for the remote phases is built on different kinds of software and methods which are introduced in Table 1 specifying for each tool what it is and how it was used. Not all elements in this table can or must be used exclusively in a remote setting, however, they can enrich both hybrid and remote settings.

Remote element	Description
Video calls	Video calls enabled us to conduct regular project meetings during completely remote phases. We used software such as Zoom (https://zoom.us/) or the open-source alternative BigBlueButton (https://bigbluebutton.org/). These calls also enabled us to invite guests to give a digital presentation or workshop on a particular topic when meetings could not be held in-person. However, video calls can also be used in-between meetings for teams as a low-threshold means to discuss and collaborate. The format was enriched by using features such as screen sharing and breakout rooms. Breakout rooms give teams their own private space to work on their project. Screen sharing furthermore enables the team to discuss code, designs and more.
Kanban boards	Kanban boards aid in structuring each team's work by formulating user stories and tasks. User stories capture the key features of a project from a particular user's point of view with details about what the feature entails and what its purpose is. From each user story, many tasks can be derived which encompass the steps for the technical implementation of a particular feature. Kanban boards give an overview about what has to be done, what is currently being worked on and which tasks and user stories are already completed. Tasks are written on cards and assigned to a column, e.g., in the most basic form, "To Do", "In Progress" and "Done". If a team member takes on as task, this member's name is written on the card and the card is moved to the "In Progress" column. The board thus also provides insight into how work is distributed among the team members. A digital realization allows access from almost everywhere and thus facilitates staying up to date with the current state of the project. Digital Kanban boards were realized using WeKan (https://wekan.github.io/) on a server hosted by us, using Trello (https://trello.com/) or the built-in Kanban boards of NextCloud (https://nextcloud.com/).

Stand-up meetings	During a stand-up meeting, students stand together either live or virtually and report to each other in a few sentences what they have done for the project since the last time, if there were any problems and what next goals they aim to achieve. In the remote version of the Digitechnikum, a "big" stand-up meeting with the whole group took place at the start of a video call while "small" stand-up meetings within a team were realized via breakout rooms after the completion of the big meeting
Git	Git is a version control software used to store current and previous versions of code to facilitate collaboration especially when multiple features are developed simultaneously. Using it in school is uncommon, however, it is essential for a long-term project such as the Digitechnikum with potentially multiple coders, who – in a remote setting – all work on different computers. Git repositories can be set up e.g. on GitHub (https://github.com/) and every team member needs a GitHub account to contribute code to the project.
Online courses	Especially at the beginning of a year when students decide on the technology stack for their project, they need additional input to learn how to use certain languages or frameworks. Fee-based Udemy courses (https://www.udemy.com/) or free YouTube tutorials enable the students to e.g. learn new programming languages at their own pace at home using instructional videos.
Cheat Sheets	Cheat sheets are pdf documents containing useful links for a certain technology or topic such as "App programming with Flutter". They include links to official documentations, installation tutorials, forums etc. Students get these pdfs to independently acquire skills at home.

Table 1: Remote elements used in the Digitechnikum.

METHODS

To discuss the elements introduced in Table 1 in more detail, we rely on both, our observations and data from the end-of-year evaluations of the first three years of the Digitechnikum with $n_1 = 22$, $n_2 = 15$, and $n_3 = 15$ participants[2], respectively, which include 5-point Likert Scale (from 1 "Do not agree at all" to 5 "Completely agree") and open text items. These evaluations are not primarily focused on remote elements but the success of the Digitechnikum as a whole containing questions about the project structure, the students' experiences, teamwork, mentoring and more. In case of prior technical knowledge, students fill out a self-assessment of their skills when applying for the Digitechnikum (first time in year 2) with the same items reappearing in the end-of-year evaluation. Should no data about a remote element be derivable from the evaluation, we critically depict our subjective experiences.

[2] Due to the Covid-related restrictions, less students could participate in the Digitechnikum from year 2 on.

RESULTS – ENHANCING COMPUTER SCIENCE PROJECTS WITH REMOTE ELEMENTS

In this chapter we describe the results of the remote conduction of the Digitechnikum during the COVID-19 pandemic with respect to each remote element from Table 1 and propose how these elements can enhance computer science projects in school, especially those with a duration of several weeks or more.

The main and most indispensable element on which remote phases of the Digitechnikum relied are video calls to facilitate coordination, exchange and team coding. Meetings, both with the whole group and within the teams of four (via breakout sessions), could only take place virtually in remote phases, even though longer remote meetings “felt strenuous at times” (quote from year 2 evaluation). Nevertheless, video calls proved to be a practical tool and a low-threshold means for quick team consultations or longer discussions even during phases in which the Digitechnikum could take place in-person. In year 3, which took place both in-person and remotely, a direct comparison of these phases was possible. Even though the mean approval rate of the statement “I liked the remote meetings” was lower with M = 3,93 (SD = 0,96) than that of the in-person meetings (M = 4,67, SD = 0,62), the acceptance of the digital phases is still satisfactory. Screen sharing as part of these video calls is a useful tool to preset and get insight into the actual code base. Moreover, it facilitates decentralized work in modern program development. Screen sharing is also excellent for so-called code reviews. Here, the code of a specific new functionality is presented and thoroughly discussed with the whole group, which contributes particularly to the teaching of programming conventions, the removal of obstructive programming habits and the fortification of the ability to take criticism. Code reviews were not used in the first three years of the Digitechnikum but are currently being tested in the fourth year due to their great potential. In sum, there are many ways in which the self-regulated use of video calls and screen sharing in a school project context can enrich students’ collaboration and communication outside of school and is, thus, worth being consciously motivated by the teacher.

Kanban boards were already an integral part of the Digitechnikum project before the COVID-19 pandemic. They belong to the selected agile methods introduced with the aim to improve teamwork and the advance of the project by highlighting values such as communication, feedback and simplicity. Aggregating the answers of all three years, the mean answer value to the statement “I think it’s helpful that we use Kanban boards.” is M = 3,52, (SD = 1,03, n = 53). The highest mean for this question was achieved in the almost all-digital run-through of the Digitechnikum in year 2 (M = 3,87, SD = 0,88), so Kanban boards seem to be at least equally helpful in a remote or hybrid setting. This impression is supported by open-text answers in year 2 to the question “What did you learn in the Digitechnikum?” where one student wrote that he used Kanban boards in the Digitechnikum for the first time and that they “are a very good tool to do work in a structured way”. Another student wrote that Kanban boards are one of the things he liked most in the project. The Kanban boards are filled with user stories and tasks (see e.g. Brichzin et al., 2019). On the Kanban board, user stories and tasks are collected in a clear and structured manner. Students can independently assign themselves to tasks and move the cards to the respective column which facilitates remote work. In spite of its positive qualities, it takes students some time to get used to this method. In the evaluation of year 1, one student writes:

"We didn't have enough time during the meetings, actually we only had enough time to distribute tasks or move cards on the Kanban board from left to right (things which could be easily accomplished at home, we thus had only little time to intensively discuss ideas within our team)."

The quote demonstrates that meetings in-person are valuable for complex discussions and briefings. Moreover, the student emphasizes that Kanban boards can also be maintained at home. As such, they offer an opportunity to bridge in-person meetings and remote phases. Overall, we observed that some teams used their Kanban boards to their full potential until the end of the Digitechnikum, while other teams acted more independently of the board over time. Thus, an obligatory introduction of the Kanban board tool followed by a voluntary continuation appears to be a worthwhile compromise in a school context.

Stand-up meetings meant to bring all team members up-to-date before the next work session. They also proved to be valuable for the other teams and the mentoring team to get insights into the presenting team's current progress. After the "big" stand-up meeting with the whole group where everyone briefly summed up what they had been working on, the meeting was continued within the project team where the next steps and goals were discussed. In the remote periods of the Digitechnikum, stand-up meetings were realized via video calls and breakout rooms. In spite of these regular stand-up meeting within the whole group, some students wished for even more knowledge exchange across teams, as a quote from the year 3 evaluation depicts: "One could exchange more with each other or stimulate this exchange by mixing the groups at the project days.". Therefore, we would recommend practicing stand-up meetings live and remotely in school projects not only to involve all teams and team members in each project's progress, but also to use this format actively as a way to exchange knowledge.

The use of Git or other version control systems is invaluable in software development, allowing team members to productively code together and to integrate work done independently into the main code base. However, using version control systems efficiently is a complex matter to teach, especially since most students even at university level have no prior knowledge about it. We try to bridge this gap with designated Git-Workshops at the beginning of the Digitechnikum. In accordance with these, the evaluations show that Git is one of the topics students learned most about during the Digitechnikum: the self-reported skill-level increased from $M = 1.87$ to $M = 3.67$ in year 2 and from $M = 2.1$ to $M = 3.73$ in year 3 with Kanban boards being the only area where the self-reported data implied a higher increase. Still, due to the complexity of the topic it could be observed that students struggled to efficiently use the system to manage their project files. For an application at school, we appeal for a thorough and practice-based excursion on the basic usage of Git first, to empower the self-regulated remote usages by the students at home.

The need for more technical input in the Digitechnikum manifests in the mean answer value $M = 3.95$ ($SD = 1.22$) to the item "I would have preferred to get more technical input" (scale from 1 "Do not agree" to 5 "Completely agree") in the evaluation of year 1. However, we did not have the personal resources to give extensive technical workshops that lasted longer than a day. Additionally, one main goal of the Digitechnikum is for students to learn how to solve problems and to learn new things independently. The necessity for a remote conduction in year 2 helped us to find a solution to this problem by using online courses (see

Table 1). As long as they are didactically reasonable and designed to fit the currently taught topic, online courses make it possible to move entire topics to remote learning, presenting themselves as an opportunity for students to learn new technologies on their own – comparable to the flipped classroom concept. These courses proved to be successful for the autodidactic acquisition of new programming languages and environments: In year 3, one student writes "I learned most new things by completing the online Flutter course". We can also report a decrease in the voiced need for more technical input from M = 3.95 in year 1 to M = 3.07 in year 2 to M = 2.94 in year 3 with the decrease from year 1 to year 2 (where the online courses were introduced first) being statistically significant (two-tailed t-test, $p = 0.05$). Yet, we observed that some teams lost focus over in-detail courses and fell behind starting with their own project implementation. To support them, we created what we call "Cheat Sheets" with important links, online course chapters and videos to find orientation when working with a new technology. The impact of these cheat sheets is currently tested in year 4.

DISCUSSION

The use of remote elements in the classroom was taken to an extreme during the COVID-19 pandemic. Distance learning as a stand-alone way of education seems not desirable from a motivational point of view as first surveys with teachers during the emergency remote teaching phase suggest (Barlovits et al., 2020, Hodges et al., 2020). Nevertheless, the sudden widespread use of digital communication tools for teaching and learning environments has also created opportunities for advancing digitization in education. In this article, we have presented the Digitechnikum, a project for the development of non-profit computer science projects by students, which had to take place entirely online for long periods of time during the pandemic. Based on our experiences, we presented remote teaching elements used during the Digitechnikum to discuss the question "How can remote teaching elements used during the COVID-19 pandemic prove worthwhile for computer science projects in the post-pandemic period?".

A variety of remote was essential for the successful implementation of the remote Digitechnikum, as well as for the development of the individual projects in the student teams. Most essential were tools supporting communication such as video call software since they enabled a remote exchange and to teach content online with the use of practical features such as breakout rooms and screen sharing. These tools also facilitate more frequent team meetings in general, and hence, their use should be encouraged also in in-school projects. The same recommendation applies for stand-up meetings both live and remotely to foster progress and knowledge exchange across the teams. In turn, Kanban boards were observed to be a concept worth teaching which was also broadly accepted by the students. Yet, they suited some teams more than others. Therefore, we root for a voluntary continuation after a mandatory introduction of this agile method. Moreover, shifting the acquisition of some relevant topics to an autodidactically remote context in combination with online courses and cheat sheets is also in line with a sensible hybrid project setting. It should be emphasized that the right choice of remote learning content is crucial to not overwhelm the students and prevent them from starting with their actual project implementation.

Considering our initial question, our experience suggests that the use of many remote elements which we put to test in the Digitechnikum can also be suitable for a post-pandemic setting in the context of school-based computer science projects. However, this does not imply a desire for conducting a pure remote project, which becomes unmistakably clear from the many comments of the type *"the Digitechnikum would have been even better in-person"* in the complete remote run-through of year 2. Therefore, we appeal for the well-dosed use of remote teaching elements to enhance the teaching and learning experience in a school-based computer science project where part of the development is meant to be conducted at home.

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THE EFFECT OF TEACHER TRAININGS ON THE ACCEPTANCE AND PERCEPTION OF THE ASYMPTOTE SYSTEM

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Abstract. This paper aims to present the first studies on an innovative technological system, ASYMPTOTE, designed after the Covid-19 pandemic to enable the teaching and learning of mathematics online. A training course (Long-Term Curriculum) on ASYMPTOTE was offered by three European institutions. The results regarding the reception of the training by the course participants are presented and their willingness to use ASYMPTOTE is investigated. The results show that the participants, both future teachers and in-service teachers, positively perceived the system and plan to integrate it into their teaching practices.

Key words: ASYMPTOTE, course experience questionnaire, technology acceptance model.

INTRODUCTION

According to the Technology Acceptance Model (TAM) (Davis, 1989), the success of integrating any innovation into education, be it related to pedagogical or technological issues, depends on teachers' attitudes and perceptions about it. Appropriate teacher training and support are crucial factors in shaping teachers' attitudes towards innovations, especially technological ones, and their intention to use them (Daher et al., 2018). Problems raised by the Covid-19 pandemic have 'forced' both teachers and students to make new use of technology, engaging them in a challenge that has seen a shift in teaching-learning from presence to online. These rapid changes inspired the Erasmus+ Strategic Partnership ASYMPTOTE. The project is being carried out by seven institutions from five European countries (Germany, Greece, Italy, Portugal, and Spain) and aims to develop an adaptive synchronous mobile system for teaching mathematics online.

This paper concerns the evaluation of the effects of a training programme designed to introduce the ASYMPTOTE system. This course, called Long-Term Curriculum (LTC, from here on), was delivered in Greece, by the University of the Aegean (UoAegean), which targeted in-service mathematics teachers. It was also addressed to future mathematics teachers in Italy, by the University of Catania (UoCatania), and in Germany, by the Goethe University of Frankfurt (UoFrankfurt). Our research aims to investigate the impact of the LTC on participants' attitudes toward the ASYMPTOTE system (about ease of learning, usefulness, attractiveness and enjoyment) and their intention to use it in the future.

In the following sections, we begin by explaining the theoretical foundations that frame the ASYMPTOTE project and the description of the technological system of the same name. We will then proceed by presenting the research methodology and data analysis after having explained our research questions. The conclusions will outline the results achieved.

Taranto, E., Barlovits, S., Fesakis, G., Triantafyllou, S., Tzioufas, N., Koutsomanoli-Filippaki, D., Volika, S., Mammana, M. F., & Ludwig, M. (2023). The Effect of Teacher Trainings on the Acceptance and Perception of the ASYMPTOTE System. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 169–176). WTM. https://doi.org/10.37626/GA9783959872522.0.20

CONCEPTUAL FRAMEWORK

Educational processes were massively changed by the Covid-19 pandemic in the Spring of 2020: Instead of taking place at a common location, namely the classroom, teaching and learning took place in student's rooms. This unforeseen phase is described by the term *emergency remote teaching (ERT)* (Hodges et al., 2020). It describes the rapid establishment of alternative forms of instruction due to the distance learning situation.

In order to design effective online courses in line with the Community of Inquiry model (Garrison et al., 2000), Salmon (2013) proposes a five-stage model that divides online instruction into phases with different requirements for technical support and moderation. It not only focuses on a) interactive and collaborative course design but also emphasizes b) the need to familiarize students with the tools used and provide technical support.

During Covid-19 distance learning, both requirements could not be fully met. Among others, Barlovits et al. (2022) identified the following five main problems in a cross-national study of ERT online instruction:

- Loss of personal interaction caused by the spatial separation of teachers and learners and the shift of instruction;
- Lack of adequate formative assessment, i.e., teachers reported issues in monitoring students' learning progress, while students perceived a lack of feedback;
- Deficit of curricular resources regarding the availability of ready-to-use materials;
- Lack of technical equipment for student's participation in online courses;
- Lack of digital competencies among teachers and learners.

THE ASYMPTOTE SYSTEM

To address these two principles of online course design, the *ASYMPTOTE system* has been developed since March 2021. It aims to provide a free, adaptive, and technically low-threshold tool for delivering online mathematics education from secondary school to university (Barlovits et al., 2022).

In ASYMPTOTE, a collection of tasks is organized in a so-called learning graph. It consists of a linear sequence of mandatory tasks (main tasks) supplemented by tasks on an easier (support tasks) or higher level (challenge tasks). As the next task is proposed based on performance in the previous task, ASYMPTOTE takes a micro-adaptive approach (cf. Plass & Pawar, 2020). At the same time, students must take responsibility for their learning process, as the support and challenge tasks remain optional. In other words, ASYMPTOTE promotes students' autonomous and self-regulated learning (cf. Greene et al., 2011) in a micro-adaptive learning environment. For a detailed account of the learning graph concept and the ASYMPTOTE system itself, we refer to Barlovits et al. (2022).

Technically, ASYMPTOTE is developed as a two-component system consisting of a web portal and a mobile app. The web portal is the teachers' workspace. Here, teachers can select available tasks or learning graphics from an open database or create their own content. In addition, teachers can follow the students' working process in real time within the Digital

Classroom feature. In addition, the Digital Classroom offers a chat (images, text or audio messages), so that teachers and learners can directly get in contact.

In the mobile app, students can work on the learning graph. It is available for iOS and Android devices. The app offers up to three hints per task and an immediate answer validation after entering a calculated solution. Further, a sample solution can be displayed.

Returning to the issues of the ERT phase mentioned above, the ASYMPTOTE system aims to address these in the following ways:

- Personal interaction: providing a chat tool for synchronous communication between instructors and learners as part of the Digital Classroom feature;
- Adequate formative assessment: providing hints, answer validation and sample solution in the ASYMPTOTE app as well as offering a real-time monitoring function for teachers within the Digital Classroom feature;
- Curricular resources: providing an open database of exemplary tasks and learning graphs and offering the possibility for teachers to create their own contents
- Technical equipment: following a mobile learning approach, as only a smartphone is required on student's side;
- Digital competencies: providing an easy-to-use mobile app for students as well as a handbook and video tutorials for teachers on how to use the web portal.

Since ASYMPTOTE has taken an adaptive and mobile approach to learn, it mostly satisfies the identified issues for online mathematics education (cf. Barlovits et al., 2022).

MOTIVATION AND RESEARCH QUESTION

This study pursues the empirical objective of analysing the effect training in ASYMPTOTE has on a group of in-service and future mathematics teachers. Specifically, we are interested, on the one hand, in analysing the participants' evaluation of their reception of the *Long-Term Curriculum (LTC)*. On the other hand, we are interested in investigating the impact of the training on the participants' attitudes towards the ASYMPTOTE system and their intention to use it. The following research questions should be answered as a result of the study:

(1) What was the reception of the LTC by its participants?

(2) What willingness did LTC participants express for using ASYMPTOTE in the future in relation to their attitude to the system?

METHODOLOGY AND DATA SOURCES

The LTC is designed to be a balanced introduction to the theoretical background of online mathematics education as well as familiarity with the use and affordances of ASYMPTOTE. It is composed of four modules, as illustrated in Table 1. Each module has assigned a specific amount of ECTS adding to 3 in total. Moreover, the last column has the in-person session in hours for each module.

#	Module	ECTS	Est. Hours
1	Theoretical background	0.5	3.5
2	Teacher's perspective with creation of own tasks	1	7
3	Student's perspective	0.5	3.5
4	Testing of a learning graph and peer reviewing of tasks	1	7
	Total	3	21

Table 1: The structure of the LTC training content.

Participants and data collection

Two groups of LTC participants are under examination in this research. The first group concerns 17 participants that attended the LTC in UoAegean while the second group includes 19 participants who attended the LTC at the UoCatania (9) and UoFrankfurt (10). The main differences among the two groups are that a) the LTC of group 1 was conducted using the blended learning model with weekly asynchronous training modules via Moodle and synchronous teleconference sessions via Zoom. The LTC for group 2 was implemented using in-person sessions. Further, b) Group 1 includes mainly in-service teachers experienced in ERT while group 2 consists of students – future math teachers without experience of ERT of mathematics. At the end of the LTC, participants were given a questionnaire. It consisted of 4 parts: i) general information; ii) specific questions on the LTC modules; iii) Course experience; and iv) Willingness to use ASYMPTOTE. The LTC questionnaire can be accessed on the ASYMPTOTE website: urly.it/3r7fv.

The items of the two included standardised questionnaires, namely the Course Experience Questionnaire (CEQ; part iii) and the Technology Acceptance Model (TAM; part iiii), were given on a 5-Point-Likert Scales.

Course Experience Questionnaire

The *Course Experience Questionnaire (CEQ)* is an instrument used as a performance indicator of teaching quality and it studies the degree of quality and satisfaction of the participants of an academic program (Ramsden, 1991; Byrne & Flood, 2003). In this research, CEQ was used to assess the quality of teaching and dimensions of the learning quality, including 21 items divided into 5 constructs: a) "Good Teaching" (GTS), b) "Clear Goals and Standard" (CGSS), c) "Appropriate Workload" (AWS), d) "Appropriate Assessment" (AAS) and e) "Generic Skills" (GSS) focussing on the skill development along the course.

Technology Acceptance Model

To model users' acceptance of digital systems, Davis (1986) proposed the *Technology Acceptance Model (TAM)*. It aims to explore the influence that external factors, such as training, can have on an individual's internal attitudes and intentions, and provides a theoretical model for predicting user acceptance of technology. The TAM Model is grounded on the assumption that the use of a digital system is determined by a) "Intention of Use" (IoU). To determine this intention, the b) "Attitude" (AT) of a person toward the specific behaviour and c) the "Perceived Usefulness" (PU) of the system by the individual are cumulatively taken into account. Moreover, a user's attitude is also related to d) the "Perceived Ease of Use" (PEU). External factors like e) "Perceived Attractiveness" (PA) of the ASYMPTOTE system, and f) "Perceived Enjoyment" (PE) of the LTC are further investigated. Detailed description of the CEQ and TAM items used per construction is available in the online version of the LTC Questionnaire.

RESULTS

The results for the CEQ and TAM questionnaires of group 1 and group 2 are presented to infer hypotheses about the impact of the LTC training on the in-service teachers (group 1) and the future teachers (group 2). The future teachers from UoCatania and UoFrankfurt can be considered as one group, since their answers to CEQ and TAM are homogenous according to the χ^2 test carried out.

Course Experience Questionnaire

Table 2 shows the descriptive statistics for CEQ including mean value μ, SD and Cronbach's α. It is notable that the construct's mean values are close to the maximum possible value. So, in general, the teachers receive the course very well in all its aspects.

CEQ			**UoAegean** GRE; N=17			**UoCatania & UoFrankfurt** ITA & GER; N=19		
Construct	*items*	*max*	*μ*	*SD (n-1)*	*Cronbach's α*	*μ*	*SD (n-1)*	*Cronbach's α*
GTS	6	30	25.24	3.65	0.77	24.89	3.41	0.74
CGSS	3/4*	15	12.71	1.45	0.60	12.47	1.50	0.65
AWS	4	20	15.00	2.29	0.59	15.21	2.66	0.69
ASS	3	15	11.12	2.76	0.66	12.11	1.85	0.66
GSS-TPK	4	20	17.00	2.09	0.77	17.21	2.18	0.74

* Item 3 for UoA CEQ and item 1 for UoC &UoF were excluded to increase Cronbach's α

Table 2: Descriptive statistics of CEQ.

However, medium reliability or agreement among the participants is shown by Cronbach's α for "Clear Goals and Standards Scale" (CGSS), "Appropriate Workload Scale" (SWS), and "Appropriate Assessment Scale" (AWS). That is considered expected because ASYMPTOTE is a new tool for the in-service and future teachers. Thus, it was not easy for participants to predict what would be their workload and their required performance. Further, the amount of workload subjective and thus varies significantly for each participant.

Technology Acceptance Model

In Table 3, the summary of the descriptive statistics for TAM is presented. All the constructs and the items have high mean values and narrow standard deviation. It seems that the teachers have a good attitude against ASYMPTOTE: they believe it is easy to learn, useful, attractive, and enjoyable so they declare the intention to use it in their classes.

TAM			**UoAegean** GRE; N=17			**UoCatania & UoFrankfurt** ITA & GER; N=19		
Construct	*items*	*max*	*μ*	*SD (n-1)*	*Cronbach's α*	*μ*	*SD (n-1)*	*Cronbach's α*
PEU	3	15	13.24	1.48	0.69	12.89	1.88	0.83
PU	3	15	11.71	1.83	0.77	12.26	2.23	0.85
PA	3	15	12.29	1.53	0.82	12.47	2.29	0.87
PE	3	15	12.71	1.53	0.83	12.68	2.14	0.92
AT	3	15	12.12	1.90	0.83	12.16	2.29	0.78
IoU	3	15	12.47	1.81	0.83	12.79	2.39	0.94

Table 3: Descriptive statistics of TAM.

For the LTC at UoAegean, we see that Cronbach's α is quite high for most of the constructs except "Perceived Ease of Use" (PEU). The latter one shows that the teachers do not have agreement on whether ASYMPTOTE is easy to use to the extent that they are with the other constructs of the TAM. On the other hand, and despite their varied opinions, it is notable that PEU shows the highest mean value of 13.24 of all six constructs. Similar is the situation to "Perceived Usefulness" (PU) where the teachers do not agree so much on questions: *PU2. I think ASYMPTOTE makes me more effective in teaching mathematics* (μ=3.82, SD=0.81), and *PU3. If I use ASYMPTOTE to teach mathematics the students will learn more easily* (μ=3.59, SD=0.80). It seems that some teachers have some doubts regarding the system's effectiveness in making teaching and learning substantially easier.

For the LTC at UoCatania and UoFrankfurt, Cronbach's α is quite high for all the constructs. The lower value of 0.78 concerns the construct "Attitude" (AT). Even if most the participants express a highly positive attitude towards ASYMPTOTE (μ=12.16, SD=2.29), they seem to disagree on item *AT3. Using ASYMPTOTE is the best way to teach mathematics in Emergency Remote Teaching* (μ=3.63, SD=0.9). Similar is the situation to PU where the future teachers do not agree so much on the question *PU3. If I use ASYMPTOTE to teach mathematics the students will learn more easily* (μ=3.79, SD=0.79). These data show that some future teachers express the same skepticism, as in-service teachers, regarding the system's effectiveness in making learning substantially easier.

Figure 1 shows the TAM model. Since all constructs are significantly correlated in pairs (Spearman's rank order correlation r, α=.05), in accordance with the TAM model, the theoretical claims of TAM are valid in the specific case of LTC participants' answers. The TAM analysis support the hypothesis that the LTC participants are possible to use ASYMPTOTE in the future and systems quality along with training had a significant impact on this result. Comparing TAM, using the data of Table 3 and Figure 1, among the two groups, mostly similar results can be observed. However, in the case of group 2 (LTC at UoCatania and UoFrankfurt) the relations between PEU and AT are not so powerful (p=0,043). So, it seems that either there was a difference in the training of the 2 groups that resulted in the future teachers having a lower PEU, or/and the difference is due to the experience that the in-service teachers had in ERT which lead to them finding the system easier to use than the students. Since ASYMPTOTE is not as complex to use the hypothesis that experience in ERT makes familiarization in ASYMPTOTE easier and results in better PEU values seems more possible.

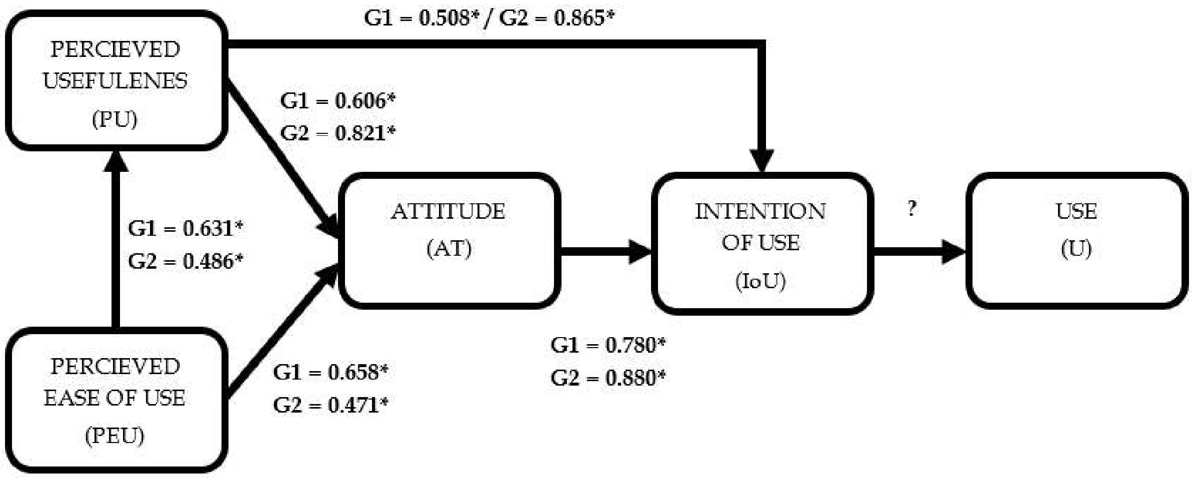

Figure 1: The TAM model verification by the LTC data for G1 (in-service teachers; UoAegean) and G2 (future teachers; UoCatania & UoFrankfurt).

Two external factors, i.e., "Perceived Attractiveness" (PA) and "Perceived Enjoyment" (PE) were included in the questionnaire. The results show that both must be considered as important factors for the acceptance of ASYMPTOTE, as they are strongly related to all theoretical constructs of the TAM (PU, PEU, AT, IoU). Overall, the LTC supported the participants to recognize the ASYMPTOTE's system advantages and formulate a positive attitude toward its integration into their educational practice. It is reasonable to support the hypotheses that ASYMPTOTE was received well by the participants as an attractive and engaging digital teaching/learning tool.

CONCLUSION

In the present study, the ASYMPTOTE system for teaching and learning mathematics online was presented during a training programme (LTC). Both, in-service teachers from Greece and future teachers from Italy and Germany participated in the course, which was evaluated by a questionnaire. In view of the *course experience* of the participants, both groups perceived the course very well in all its aspects (see CEQ). Regarding their *willingness to use the system* in the future (see TAM), we can observe that both groups showed a positive attitude. One difference between the two groups can be found in their perception of how easy to use is the system. It is plausible to assume that this difference in attitude is to be found in their prior teaching experience. In fact, while in-service teachers have also worked during the ERT, future teachers have not yet been able to gain any concrete teaching experience. Another possible explanation for this finding might be grounded in the different availability of ready-to-use resources between the three countries. It can be assumed that there is a larger number of available learning platforms and materials in Italian or German than in Greece. Regarding the intellectual merit of the project, we would like to emphasise how the ASYMTPTOTE system and in particular these first training courses for (future) teachers have impacted on the mathematical knowledge of the participants in terms of how they can integrate a new technology into their teaching practices in order to facilitate online mathematics learning. In addition, this work contributes in general to the design of the evaluation of teachers' education and training for using digital technology in education. A limitation of this study is certainly the small and heterogeneous sample to which the LTC was proposed. Furthermore, both groups constitute small numbers to be able to consider the statistics obtained as general. However, these results are the first obtained in terms of knowledge and readiness to use ASYMPTOTE. Since the system is still being developed, further studies need to be conducted on how in-service and future teachers experience the system after its further developments.

Acknowledgment

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POSTERS

TEACHING MATHEMATICS WITH ROBOTICS – A STEM PROJECT

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Abstract. In an engineering school, mathematics applications are diverse and the ISEP (School of Engineering, Polytechnic of Porto) laboratories are the ideal setting to show just that. In this article we will present the project *Matemática, por onde andas?* (Mathematics, where are you?). This project is aimed at secondary school students and intends to show that mathematics is transversal to different areas. With the help of technology, mathematics is applied in engineering projects, and in this work we will present an application to robotics.

Key words: Robotics, STEM, technology.

INTRODUCTION

In September 2014, the *Matemática, por, onde andas?* (MPOA) project was born at ISEP. Each session lasts 2 hours and is directed to secondary school students. The MPOA project intends to bring an interdisciplinary approach building a more dynamic and interesting teaching and learning process. Considering the close relationship between mathematics and engineering, the MPOA project aims to show young students that mathematics is essential in technical fields such as engineering. Mathematics intervenes in all areas of engineering, therefore, engineering can be a mean to show the applicability of mathematics and to awaken in students the interest and motivation for its learning. The Accreditation Board for Engineering and Technology (ABET), in its EAC Criteria for 1999-2000, of 1998, highlights the importance and the use of various branches of mathematics in engineering.

The advantages of interdisciplinary studies are widely accepted by teachers and researchers. Some authors refer that interdisciplinary works use knowledge that comes from different curriculum areas that offer different perspectives on a particular problem, making the curriculum more compact and more consistent. Furthermore, interdisciplinary works provide the students with relevant, challenging and enjoyable learning experiences (Clark & Wallace, 2015; Chettiparamb, 2011; Schmidt, 2015; Păvăloiu, Petrescu & Dragomirescu, 2015).

THE *MATEMÁTICA, POR ONDE ANDAS?* PROJECT

MPOA project intends to bring an interdisciplinary approach building a more dynamic and interesting knowledge process. It is intended to show to the students that mathematics is essential and an important tool, in technical fields like engineering. Making use of appealing area, such as robotics, the students witness the applicability of mathematics in engineering projects (Caldeira et al, 2016).

Caldeira, A., Faria, A., Barbosa, R., Gavina, A., Figueiredo, I., & Figueiredo, L. (2023). Teaching Mathematics with Robotics – A STEM Project. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 179–182). WTM. https://doi.org/10.37626/GA9783959872522.0.21

Project stages

Stage 1: Welcome students. Breaking the ice by a small talk to the students (so that they feel more comfortable) and an introduction to the theme (15 minutes).

Stage 2: Case Study: Mathematics in Robotics (25 minutes). In this stage, mathematical knowledge is crossed with engineering, remembering, and showing the mathematical concepts involved (such as geometry, linear velocity and angular velocity) in order to be able to control a robot (Figure 1).

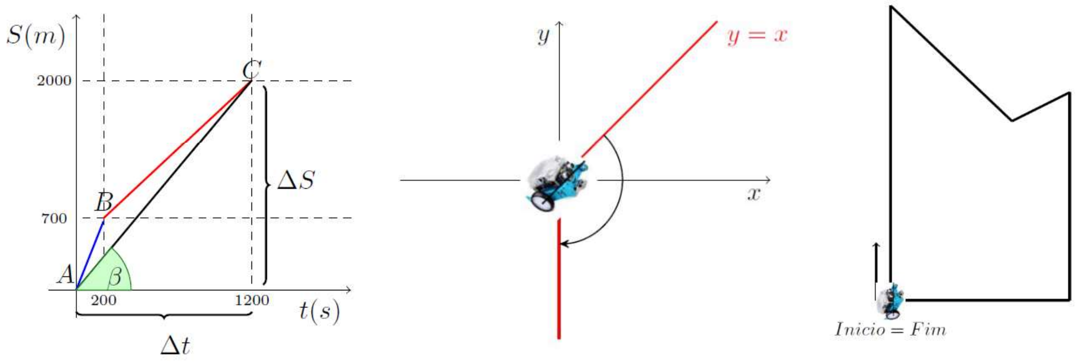

Figure 1: Some mathematical concepts involved to control a robot.

Stage 3: Introduction to block programming (20 minutes). To program a robot like the Mbot (https://www.makeblock.com/steam-kits/mbot), the mBlock graphical programming environment is used.

- Allows you to program robots and games.
- The mBlock interface is very friendly and intuitive.
- The programming language consists of the use of blocks, previously defined, which allows "giving orders" to the robot.

Stage 4: Activities in the robotics laboratory - programming and simulation of paths (60 minutes). Hands-on the robot, trip to the ISEP engineering laboratories. The robot is programmed to run a certain path and tested on the track. The students test their mathematical knowledge in engineering projects – applied knowledge.

Results

In the academic year 2021/2022, 14 teachers and 151 students participated in the MPOA project. A survey was carried out among teachers, to obtain their opinion on the project. The results are very satisfactory and motivating for the continuation of the project. We can see that most of the teachers are very satisfied in all points of the survey. The only exception is related to the impact on student motivation for learning, where 88% of the teachers answered that they were very satisfied, with 12% of the teachers that they were satisfied. The results are shown in Table 1. In addition, teachers were asked to mention the strengths and weaknesses of the project. It stands out as strong points mentioned: active participation of the students, the theme, the pedagogical strategy and the transversality of the contents. Only one weak point was mentioned: the duration of the session being short.

Inquiry questions	Satisfied	Very satisfied
Purpose and nature of the project		100%
Adequation of the MPOA to the school's educational project		100%
Activity structure		100%
Student satisfaction and engagement		100%
Impact on student motivation for learning	12%	88%
Overall appreciation of the project		100%

Table 1: Results from inquiry to teachers

The students reveal interest, motivation and curiosity when they were given the possibility of using mathematics in engineering projects. Through real problem situations, we emphasize the relevance of developing several skills and attitudes essential to achieve results, showing the importance of mathematics in engineering.

CONCLUSIONS

Making use of appealing areas, such as robotics, the students witness the applicability of mathematics in engineering projects. The perception of importance of mathematics in building a successful academic path is crucial for the teaching and learning process. The project "Mathematics, where are you?" Aims to motivate students early in this direction. The results obtained with specific sessions for audiences showed that are well received by students and meet the proposed objective. Through real problem situations we show the importance of Mathematics in Engineering. As future work, we intend to carry out a study to assess students' opinions about MPOA and what impact had on their math learning.

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Teaching Mathematics with Robotics – a STEM project

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Introduction

In September 2014, the "*Matemática, por, onde andas?*" (MPOA) project was born at ISEP.
MPOA is directed to secondary school students and consists of a partnership between mathematics and engineering.
This project intends to bring an interdisciplinary approach building a more dynamic and interesting teaching and learning process.
It is intended to show to the students that mathematics is essential and an important tool, in technical fields like engineering.

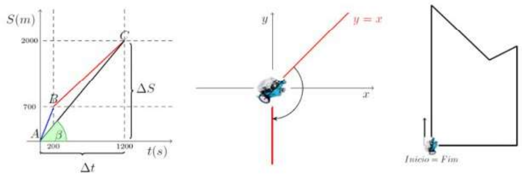


Figure 1: Some mathematical concepts involved in robotics

Objective

Dissemination of activities involving the application of mathematics in engineering.
Bring an interdisciplinary approach building a more dynamic and interesting knowledge process.
Show to the students that mathematics is essential and an important tool, in technical fields like engineering.
Making use of appealing area, such as robotics, the students witness the applicability of mathematics in engineering projects.

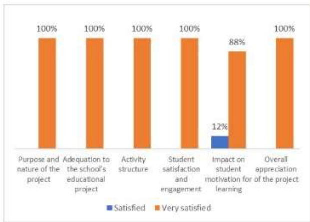
Project stages

Stage 1: Welcome students. Breaking the ice by a small talk to the students (so that they feel more comfortable) and an introduction to the theme (15 minutes);
Stage 2: Case Study: Mathematics in Robotics (25 minutes);
Stage 3: Introduction to block programming (20 minutes);
Stage 4: Activities in the robotics laboratory – programming and simulation of paths (60 minutes).

Results

A survey was carried out among teachers, in the 2021/2022 school year, to obtain their opinion on the project.

Results from inquiry to teachers



In addition, teachers were asked to mention the strengths and weaknesses of the project. It stands out as strong points mentioned: active participation of the students, the theme, the pedagogical strategy and the transversality of the contents. Only one weak point was mentioned: the duration of the session being short.

Analysis

Engineering can be a mean to show the applicability of mathematics and to awaken in students the interest and motivation for its learning.
MPOA project allows Students have the opportunity to applicate mathematics concepts with robotics and test using a STEM coding robot.
The robot is programmed to run a certain path illustrated in Figure 1 and tested on the track.

Conclusions

The students reveal interest, motivation and curiosity when they were given the possibility of using mathematics in engineering projects.

Through real problem situations, the importance of developing several skills and attitudes essential to achieve results have been shown.

Through real problem situations, was shown the importance of mathematics in engineering.

Related Literature

Caldeira, A. Faria, A., Brás, H., Sousa, A. (2016). Integração no Ensino Superior – a Matemática na Engenharia. In proceedings book, Congresso Nacional de Práticas Pedagógicas do Ensino Superior 2026 (CNaPPES2016), Julho de 2016, Lisboa, Portugal. ISBN 978-989-98576-5-0

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THE ASYMPTOTE PROJECT: DEVELOPING AN ADAPTIVE AND SYNCHRONOUS LEARNING PLATFORM

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Abstract. The ASYMPTOTE system enables teachers to deliver adaptive and synchronous mathematics lessons. In doing so, teachers can select or create so-called learning graphs in a web portal. Via code, students can download these learning graphs and work on them in a self-guided manner. In the following, the ASYMPTOTE web portal and app as well as the so-called Digital Classroom feature are briefly introduced. In addition, we introduce the Erasmus+ Strategic Partnership ASYMPTOTE, under which the system of the same name is developed.

Key words: Adaptivity, ASYMPTOTE, mobile learning.

THE ASYMPTOTE SYSTEM

ASYMPTOTE aims at the delivery of adaptive and synchronous mathematics education. Following a *mobile learning approach*, only a smartphone is required on student's side to participate in online lessons conducted with ASYMPTOTE. For teachers, ASYMPTOTE offers a web portal allowing the selection of available or the creation of own learning contents, i.e., tasks or *learning graphs* (LG). With the Digital Classroom feature (part of the web portal) a synchronous monitoring tool is available.

The concept of learning graphs

In ASYMPTOTE, a LG consists of a linear sequence of mandatory tasks (main tasks, yellow) which cover the expected learning level. They are supplemented by related tasks at an easier (support tasks, green) or higher level (challenge tasks, purple). An exemplary LG is presented in Figure 1 (left). Possible ways of accessing tasks are indicated by the arrows.

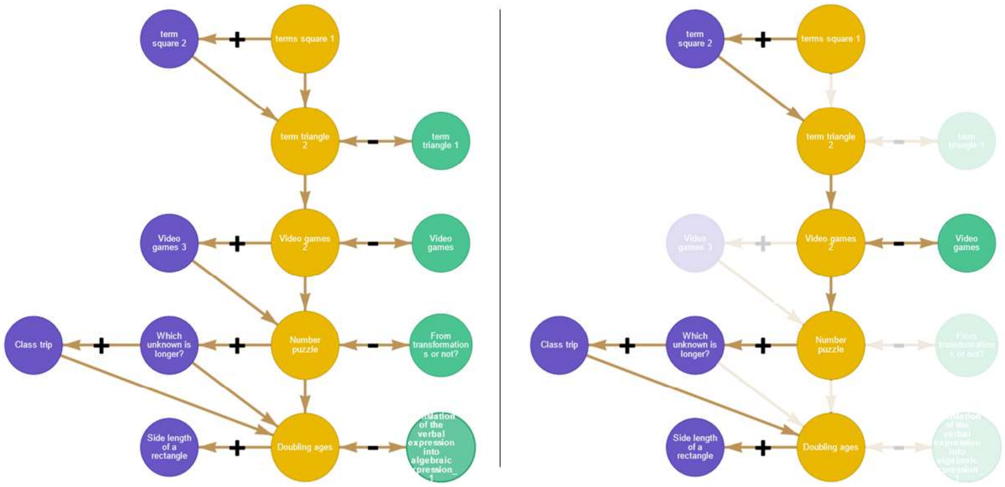

Figure 1: The LG "Reasoning and modeling with linear equations" (left) and an exemplary work process on it (right).

Oehler, D.-X. K., Anhalt, L., Barlovits, S., Ludwig, M., & Kleine, M. (2023). The ASYMPTOTE Project: Developing an Adaptive and Synchronous Learning Platform. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 183–186). WTM. https://doi.org/10.37626/GA9783959872522.0.22

Students are advised to work on support tasks if they enter a wrong solution twice in the related main task. Thus, ASYMPTOTE follows a *micro-adaptive approach*, since the next task is proposed based on performance in the previous task (cf. Plass & Pawar, 2020). The challenge tasks are optional after solving the corresponding main task. Hence, the LG concept also fosters students' autonomous and self-regulated learning (cf. Greene et al., 2011).

This is illustrated by the work process of a student shown in Figure 1 (right). After solving the first main task, the student works on a challenge task. The next main task can also be solved, while the third main task can only be solved after the associated support task has been worked on. Afterwards, the student is again able to solve the fourth and the fifth main task and additionally access three related challenge tasks. For a more detailed description of the LG concept, we refer to Barlovits et al. (2022).

The web portal

The ASYMPTOTE web portal allows (a) to create own tasks and LGs tailoring the learning content to the individual needs of the learning group. In addition, teachers can (b) select tasks and LGs from the community-based ASYMPTOTE database – a pool of tasks and LGs related to different areas of mathematics in all languages of the project partners (see final section).

The smartphone app

The smartphone app represents the students' view on ASYMPTOTE. Within the app, the learners work on LGs in their own pace. Teachers provide their students access to LGs via a code, which has to be entered in the app. In Figure 2, an exemplary working progress of a student is displayed.

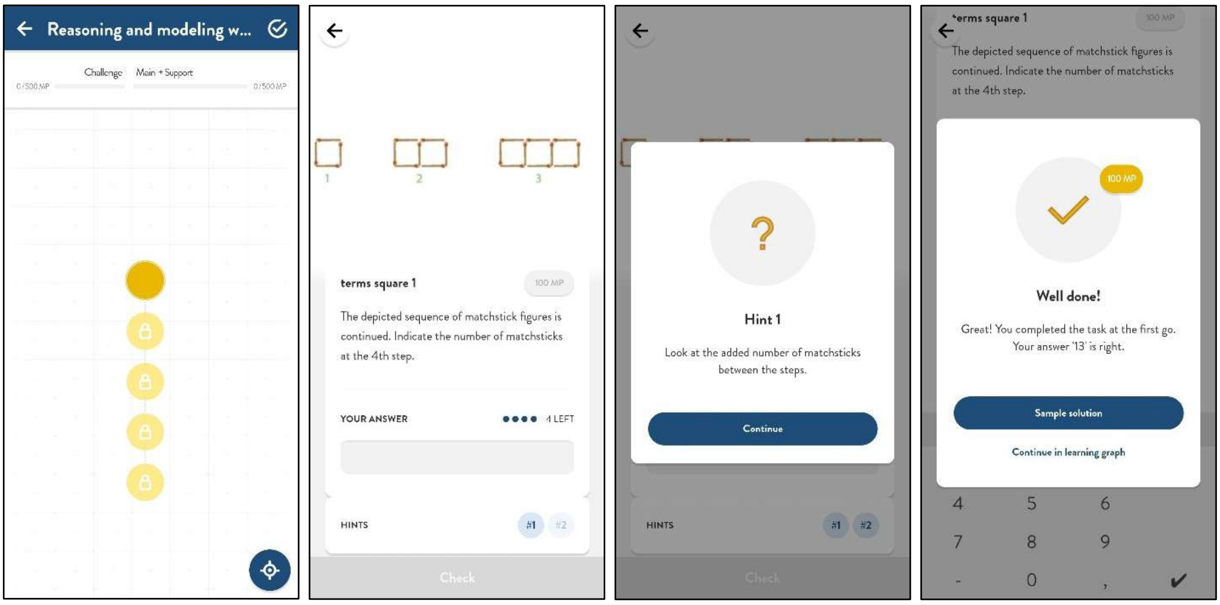

Figure 2: Students' view of the ASMYPTOTE app: accessing a LG and a task including hints and answer validation (from left to right).

A LG starts with the first main task. Selecting this task opens the task view consisting of a task image, a task formulation and an answer field for entering the calculated solution. In addition, hints are provided for the students in order to guide their work process. After the learners enter their calculated solution, the answer will be directly validated by the app. Furthermore, a sample solution can be viewed.

After completing a LG, students receive an individual summary of their work process. They can restart the LG at any time to process the LG again, e.g., for repeating the topic.

The Digital Classroom

The Digital Classroom feature is the third component of ASYMPTOTE. It provides a direct link between the web portal and the smartphone app for *synchronous* online sessions. While the students work on the chosen LG, the teacher can monitor their progress in real time. Therefore, a class overview on all students and an individual event-log for retracing the individual work process is offered. In addition, the Digital Classroom includes a *teacher-student chat* to allow a direct teacher-student interaction. Hence, the Digital Classroom feature makes the ASYMPTOTE system applicable not only for homework or exam preparation but also for online lessons and blended learning phasis.

THE ASYMPTOTE ERASMUS+ PROJECT

The ASYMPTOTE system is developed within an Erasmus+ Strategic Partnership (DAAD; grant no. 2020-1-DE01-KA226-HE-005738) of the same name. Hereby, ASYMPTOTE stands for “Adaptive Synchronous Mathematics Learning PaThs for Online Teaching in Europe”. It is carried out by seven institutions from Germany (Goethe University, University of Bielefeld & Autentek GmbH), Greece (University of the Aegean), Italy (University of Catania), Portugal (Polytechnic Institute of Porto) and Spain (teacher federation FESPM). Within the project, six Intellectual Outputs are pursuit to enrich the technical development with exemplary learning content, a long-term curriculum, scientific evaluation, and dissemination activities. The latter includes an 11-day intensive study programme for teacher students (Sep. 2022), an international teacher training (Oct. 2022), as well as a massive open online course (MOOC) for teacher students and in-service-teachers (Oct.-Dec. 2022).

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asymptote

Developing An Adaptive And Synchronous Learning Platform

Co-funded by the Erasmus+ Programme of the European Union

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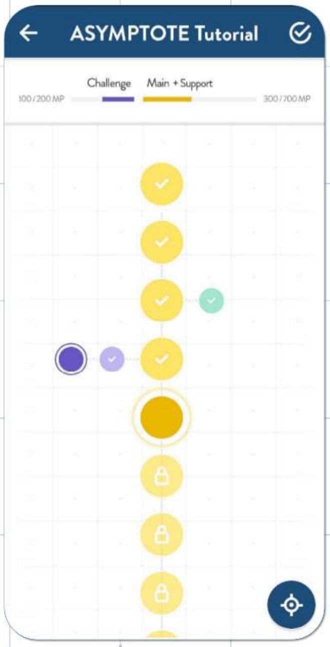
Deng-Xin Ken Oehler[1], Léon Anhalt[2], Simon Barlovits[1], Matthias Ludwig[1], Michael Kleine[2]

[1]Goethe University Frankfurt, [2]University Bielefeld

ASYMPTOTE stands for *Adaptive Synchronous Mathematics Learning Paths for Online Learning in Europe.*
It aims to deliver adaptive and synchronous mathematics lessons via so-called **learning graphs**.



Erasmus+ Strategic Partnership

DAAD; 2020-1-DE01-KA226-HE-005738

Consortium

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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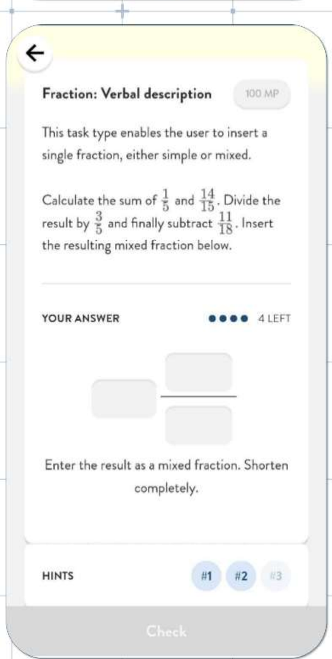
Erasmus+ Intellectual Outputs (IO):

- IO1 – Web Portal
- IO2 – App
- IO3 – Generic Tasks
- IO4 – Generic Tasks, Handbook
- IO5 – Long-term Curriculum, MOOC
- IO6 – Research, Validation



Features

✓ Adaptivity through challenge & support tasks
✓ Synchronicity via the Digital Classroom
✓ Multilingual open task pool
✓ International teacher community

Contact

Deng-Xin Ken Oehler (*oehler@math.uni-frankfurt.de*)

www.asymptote-project.eu

WORKSHOPS

THE ASYMPTOTE PROJECT: DEVELOPING A SYSTEM FOR ADAPTIVE AND SYNCHRONOUS ONLINE LEARNING

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Abstract. Within the ASYMPTOTE project, a system of the same name has been developed to prepare, conduct, and evaluate mathematics lessons online. It consists of three components, namely a web portal, a mobile app, and the Digital Classroom feature. In the web portal, teachers can select available or create own learning contents. These learning contents can be downloaded by the students to their smartphones by entering a related code in the app. Within the Digital Classroom feature, synchronous mathematics lessons can be carried out – it offers a monitoring tool allowing teacher to retrace students' working processes. In addition, a chat function is integrated for a direct teacher-student interaction. In the workshop, the participants get to know the ASYMPTOTE system and explore it from the student's and teacher's side. In the following, the ASYMPTOTE web portal and app are briefly introduced. In addition, the Erasmus+ project's outputs are presented.

Key words: ASYMPTOTE, mobile learning, online education.

THE ASYMPTOTE PROJECT

The Covid-19-induced school lockdowns all over the world in Spring 2020 led to a phase of *Emergency Remote Teaching* (ERT). The term describes the urgent and overwhelming installment of new ways to teach and to learn due to the distance situation (Hodges et al., 2020). Based on these experiences and on theoretical considerations on online teaching and learning, the ASYMPTOTE project has been started. It aims at the development of a low-barrier and adaptive learning environment for synchronous mathematics education.

The system with its three components, namely the web portal, the app and the Digital Classroom feature, are presented in the following. For more information on the theoretical background of ASYMPTOTE, we refer to Barlovits et al. (2022).

THE ASYMPTOTE SYSTEM

First Component: The web portal

The web portal (available at https://asymptote-project.eu/en/portal/) serves as a working environment for teachers to select and/or create ASYMPTOTE learning content in the form of adaptive learning graphs. These learning graphs are composed of mathematical tasks that are created in the web portal, too. An exemplary learning graph is presented in Figure 1. By working on the mandatory main tasks (yellow), the students should get a good overview of the entirety of the respective topic. The optional challenge tasks (purple) extend the main tasks with more demanding questions for the quick and particularly motivated students. In addition, support tasks (green) can be attached to each main task to give students easier step-by-step tasks. The students can access them if needed. By this structure, the learning graph concept offers a micro-adaptive learning environment and simultaneously respect the

Barlovits, S., Oehler, D.-X. K., Jablonski, S., & Ludwig, M. (2023). The ASYMPTOTE Project: Developing a System for Adaptive and Synchronous Online Learning. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 189–192). WTM. https://doi.org/10.37626/GA9783959872522.0.23

value of autonomous learning, since students must take responsibility for their learning process (cf. Barlovits et al., 2022).

Each task consists of a problem definition, an answer format (e.g., exact values, fill-in-the-blanks, or multiple choice), a sample solution and up to three stepped hints. In addition, a picture can be given. All of these task components allow students in the follow-up to work on the task independently and with automatic answer validation in the ASYMPTOTE app (cf. "Second Component"). The web portal also offers the possibility of publishing tasks and learning graphs to make them available to the ASYMPTOTE community. Hereby, it is mandatory to receive a review. In return, teachers can take advantage of the constantly growing collection of high-quality tasks. Even more, they can benefit from high-quality content from other European countries, as both tasks and learning graphs are translatable. In course of one of the intellectual outputs of the Erasmus+ project ASYMPTOTE, a solid foundation of theme-based learning graphs has been predesigned, iteratively reviewed and translated into all partner languages, which serve as a good starting point for new users.

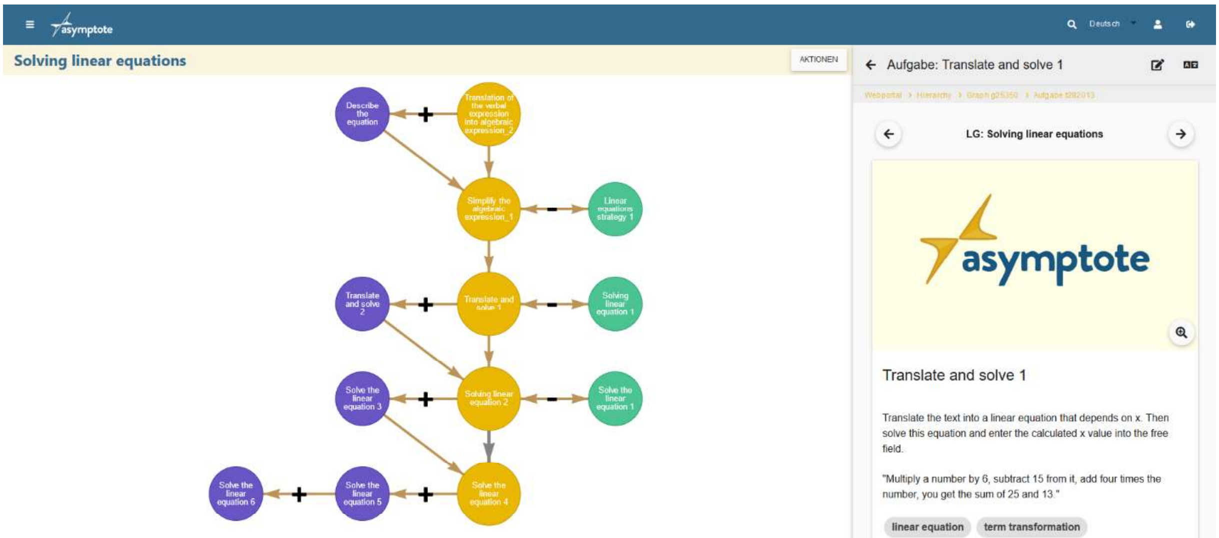

Figure 1: Web portal view of the task "Translate and solve 1" in the learning graph "Solving linear equations" with its structure on the left side.

Second Component: The smartphone app

The app enables students to work on the learning graphs in a technically low-key, self-regulated manner. By entering a code, they are connected to the learning graph to be worked on and can then solve as many tasks as possible at their own pace. An exemplary task solving process in the ASYMPTOTE app is shown in Figure 2.

It is up to the individual student whether support tasks are consulted, or a challenge task is tried if the students previously succeeded in the main task. Students have four attempts per task, the first of which is a free attempt in case of an accidental wrong entry. In case of incorrect answers, students are encouraged to use hints, or solve a support task firstly in case one is provided by the teacher. The student's input is immediately validated by the app, i.e., the students receive feedback on the correctness of the task. Moreover, if provided, the student can directly choose to continue working on a challenge task. Also, the sample solution can be viewed to validate and retrace the calculation steps once a task is completed. On a motivational level, the ASYMPTOTE app uses elements of shallow gamification (cf. Lieberoth, 2015) in the form of points, which can be viewed by the students at any time.

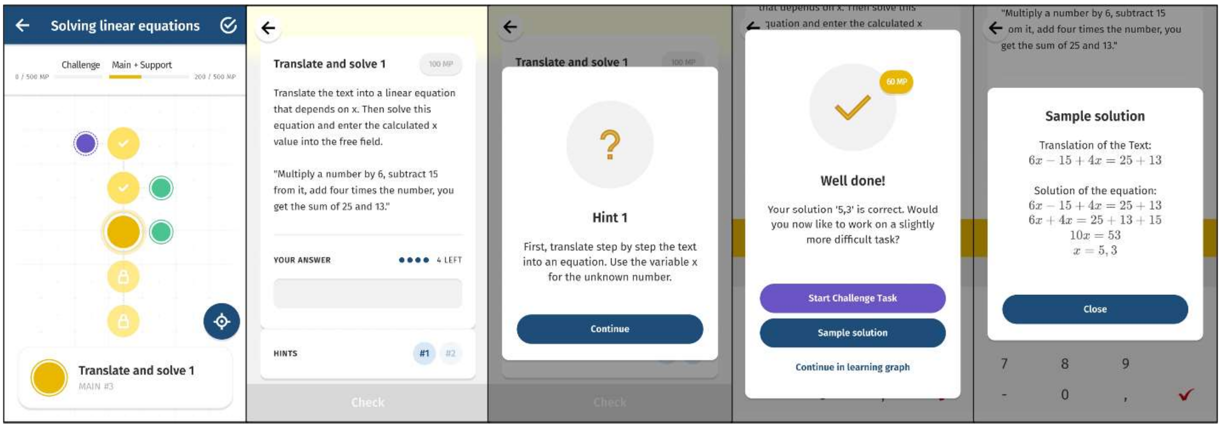

Figure 2: An exemplary task solving process in the ASYMPTOTE app: opening the task, problem description, hints, answer validation and sample solution (from left to right).

Third Component: The Digital Classroom

The Digital Classroom enables the teacher to orchestrate the processing of a learning graph in real time and to assist students synchronously in case of problems. This feature is characterized by the following core features (see also Figure 3):

- Monitoring tool: In order to respond quickly to problems, as well as provide in-depth evaluation of learning graph processes later, the teacher can track all of each student's interactions with the app. This includes typed answers, viewing hints, and viewing the sample solution.
- Evaluation: With the data from progress tracking, there are many opportunities to analyze the learning activity through the learning graph in more depth and/or optimize future learning graph processes based on that.
- Chat function: The teacher is able to send messages to all students or only to individuals through the Digital Classroom. In turn, students can contact the teacher via the app with questions or difficulties, sending pictures or voice messages.

The Digital Classroom is currently under active development to add further functionalities. For example, in the new Digital Classroom, only one QR code will be issued to students for connection. Teachers will be able to represent, manage and analyze their classes in the portal and it will be possible to edit more than one learning graph per Digital Classroom. This will also provide the basis for a number of advanced analysis tools for the teacher that will not only improve the simple evaluation of a learning graph process but will also allow for long-term analysis. The further developed Digital Classroom will be available at the conference.

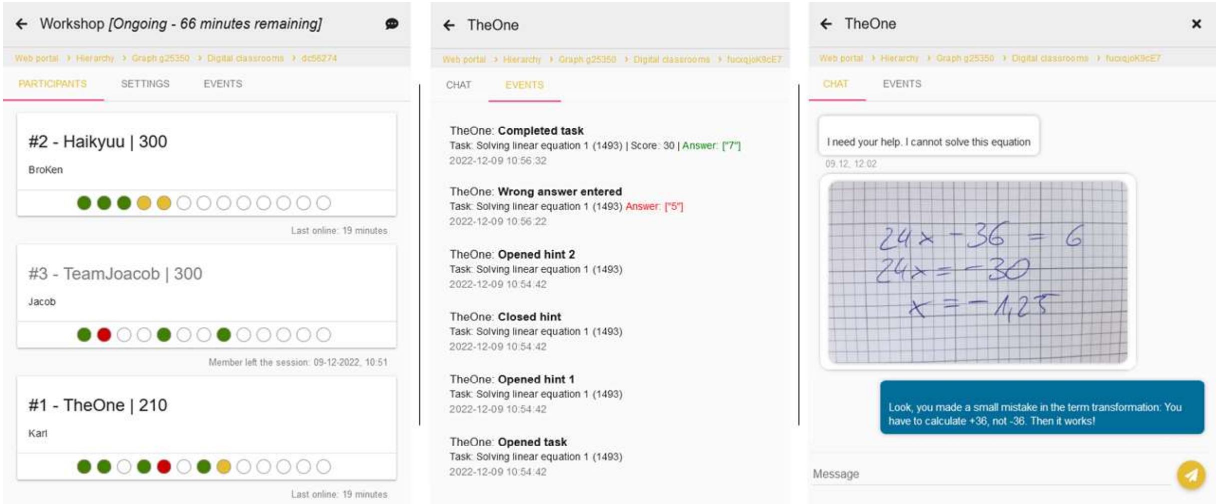

Figure 3: The Digital Classroom feature: monitoring, evaluation and chat (from left to right).

OUTLOOK: THE WORKSHOP

In the workshop, the participants get to know the system both from a student's and a teacher's perspective. Hereby, the participants work on a learning graph via the ASYMPTOTE app. Subsequently, the participants can share their experiences on how to use the app and discuss on desirable further developments. The teacher's perspective is focused on in the second part of the workshop. It is presented how to create tasks and learning graphs in the web portal, as well as how to use the Digital Classroom feature. The participants create their own tasks in the ASYMPTOTE web portal. The workshop ends with a discussion on the potentials and limitations of ASYMPTOTE.

To participate in the workshop, the ASYMPTOTE app should be downloaded beforehand. Also, a registration in the ASYMPTOTE web portal is needed. All participants should bring their own laptops and have a smartphone/tablet on site. Let's join the world of ASYMPTOTE!

Acknowledgment

The ASYMPTOTE project is co-funded by the European Union as part of the Erasmus+ Strategic Partnership Programme (DAAD; grant no. 2020-1-DE01-KA226-HE-005738).

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MOBILE LEARNING OUTSIDE THE CLASSROOM WITH MATHCITYMAP

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Abstract. Discovering mathematics in one's own environment inspires students. Numerous everyday objects offer potentials for posing interesting and motivating measuring tasks. To organize the mathematics lesson outdoors, teachers can create so-called math trails in their environment using the MathCityMap system: a web portal and a smartphone app are available to prepare and conduct a math trail. Equipped with the app and measuring tools, the students work in small groups on the object side. Hereby, the MathCityMap app supports the independent and collaborative learning of students offering hints and direct answer validation. In the following paper, the components of MathCityMap are presented.

Key words: MathCityMap, math trails, mobile learning.

MATH TRAILS AS AN INTERACTIVE LEARNING FORMAT

Our environment is full of mathematics. Such experiences can be realized with the use of math trails focusing on mathematical problems about real-life objects (Shoaf et al., 2004). Equipped by a map indicating where the tasks are located and measuring tools such as a folding ruler and measuring tape, students work on these mathematical problems outdoors. The goal of a math trails becomes obvious: doing math actively on site of an object and to discover mathematics in the real world (cf. Shoaf et al., 2004).

In an educational context, students can be sent on a theme-based trail in small groups (Barlovits et al., 2020). The tasks or stations of a math trail are completed at an individual pace and by means of mathematical activities, i.e. measuring, calculating and counting. This collaborative and interactive character can be supported by the use of digital tools: The MathCityMap system uses two technical components - a web portal and a smartphone app. The web portal supports the preparation, implementation and follow-up activities of math trails from a teacher's perspective. The use of the smartphone targets the reality of students' digital lives and supports them in their individual working and solution process (see in addition Ludwig & Jablonski, 2021).

THE MATHCITYMAP SYSTEM

First Component: The web portal

In the MathCityMap web portal (https://mathcitymap.eu/de/portal/; see Figure 1), teachers can select contents from a database of over 1.700 public math trails and 15.500 tasks worldwide. Furthermore, own tasks and math trails – adapted for the needs of the individual learning group – can be created.

Jablonski, S., Barlovits, S., Gurjanow, I., Larmann, P., Ludwig, M., Oehler, D.-X. K., & Wetzel, S. (2023). Mobile Learning Outside the Classroom with MathCityMap. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 193–196). WTM. https://doi.org/10.37626/GA9783959872522.0.24

The tasks are positioned at the location of the object with the help of GPS data which can be extracted automatically from an uploaded image. Besides an image of the object, a complete task comprises hints that students can call up when needed, a sample solution, as well as required measuring tools. In order to be able to validate the solution immediately, the answer formats multiple-choice, interval (correct, mediocre or wrong range) exact solution, vector and fill-in-the-banks are available. Especially in measurement tasks, where small inaccuracies in the measurement should not lead to a wrong result, the interval is recommended as a solution format. With the help of support tasks, complex tasks can be broken down into smaller task units. If, for example, several measurements and calculations are necessary to solve a task, the learner can be guided in small steps and each step is validated separately. The teacher can decide whether the support tasks are mandatory or optional.

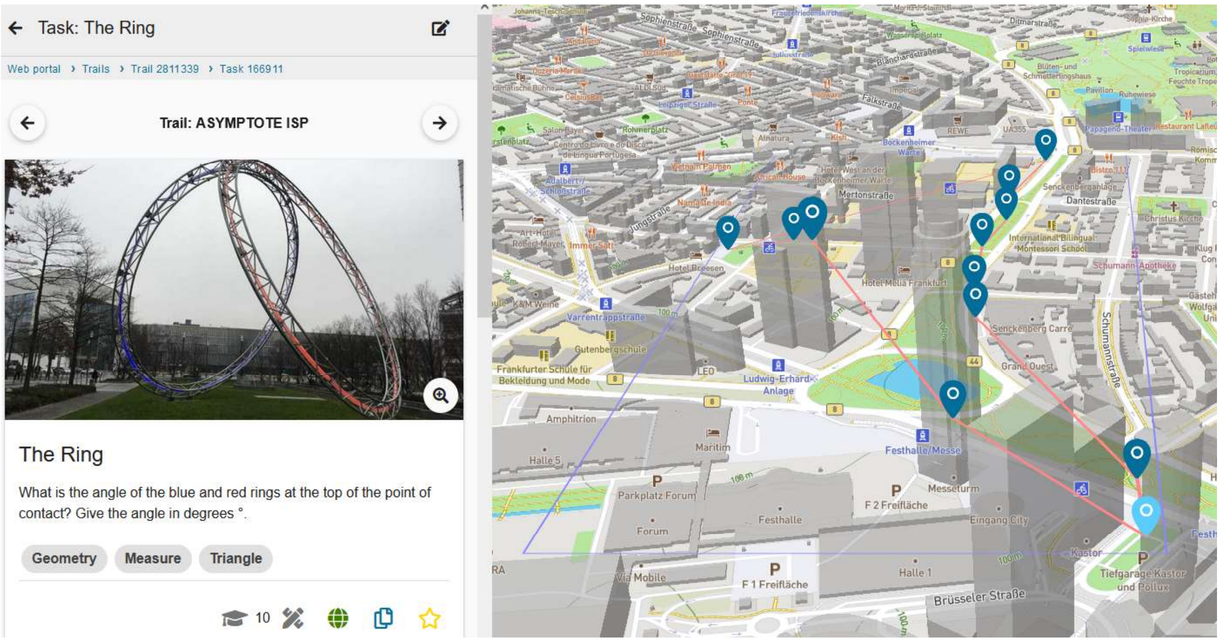

Figure 1: The task "The Ring", which is part of a math trail, as viewed in the MathCityMap web portal.

Second Component: The smartphone app

The corresponding smartphone app MathCityMap is freely available for Android and iOS (see Figure 2). It supports the students in working on a math trail created by the teacher in the web portal. The app shows the student's own position and the location of the tasks on a map. Furthermore, the tasks previously formulated by the teacher can be called up, including the hints. For the answer validation, the app gives an immediate feedback on an entered result. In addition, students can view the sample solution after ending a task. After completing a task, the app automatically leads a student to the next task.

All functionalities of the app combined support the independent work of small groups while completing a math trail. In particular, the fact that the teacher cannot be present at all math trail tasks at the same time makes the necessity of the app as a digital support tool clear. In order for the teacher to keep track of what is going on, the Digital Classroom feature of MathCityMap provides additional support.

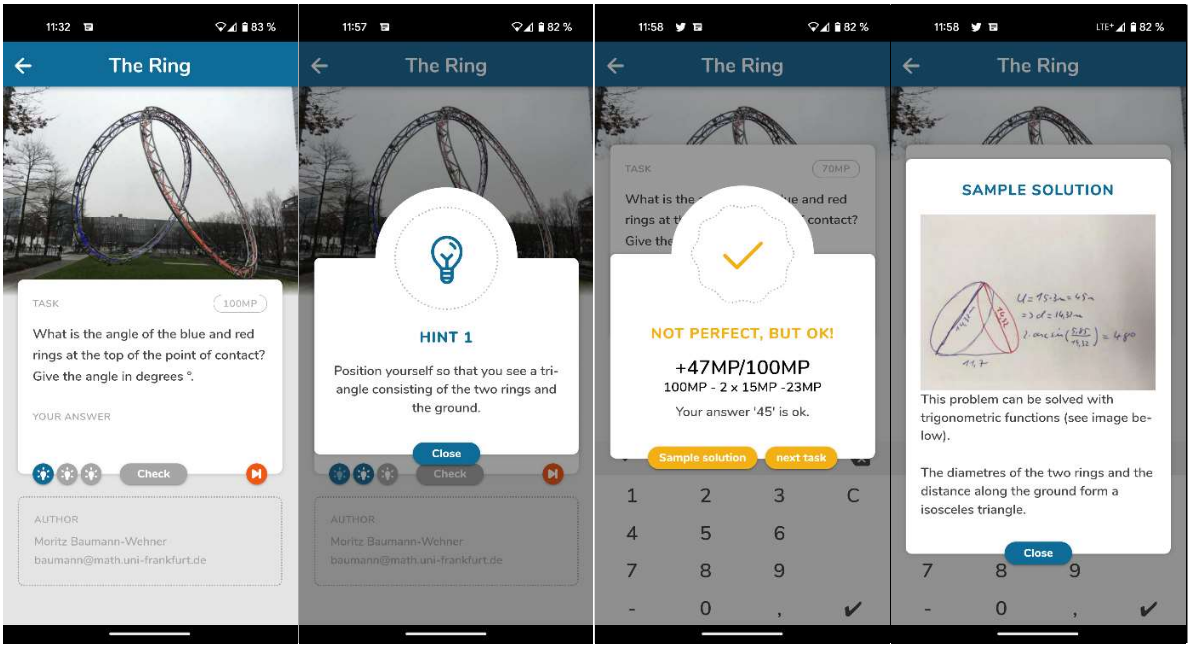

Figure 2: The task “The Ring” displayed in the MathCityMap app: task formulation, hints, immediate app feedback and sample solution (screens from left to right).

Third Component: The Digital Classroom

Within the so-called Digital Classroom, an optional and temporary digital learning environment in the web portal, the teacher can use the following additional functions to orchestrate the students’ working process on a math trail:

- Chat function: The teacher can communicate with selected groups or all students in case of queries and problems. The groups can also write to the teacher. Hereby, it is possible to send text, images and audio recordings.
- Tracking Tool: In this section, the teacher can track the progress of each group as they walk the trail, see their current positions and walking paths as well as their input on individual tasks (see Figure 3).
- E-portfolio: The entered results of each group are still available afterwards. From this, procedures and also error types can be identified which can be used for diagnostic measures and for further follow-up activities.

The Digital Classroom also supports general organization by automatically assigning a starting task to each group, if required. If the number of tasks is twice as large as the number of small groups, the system ensures that one task is placed between each two groups to accommodate different work places. In general, no personal data of the students is stored and processed when using the Digital Classroom, so that the tool is in line with European data protection standards.

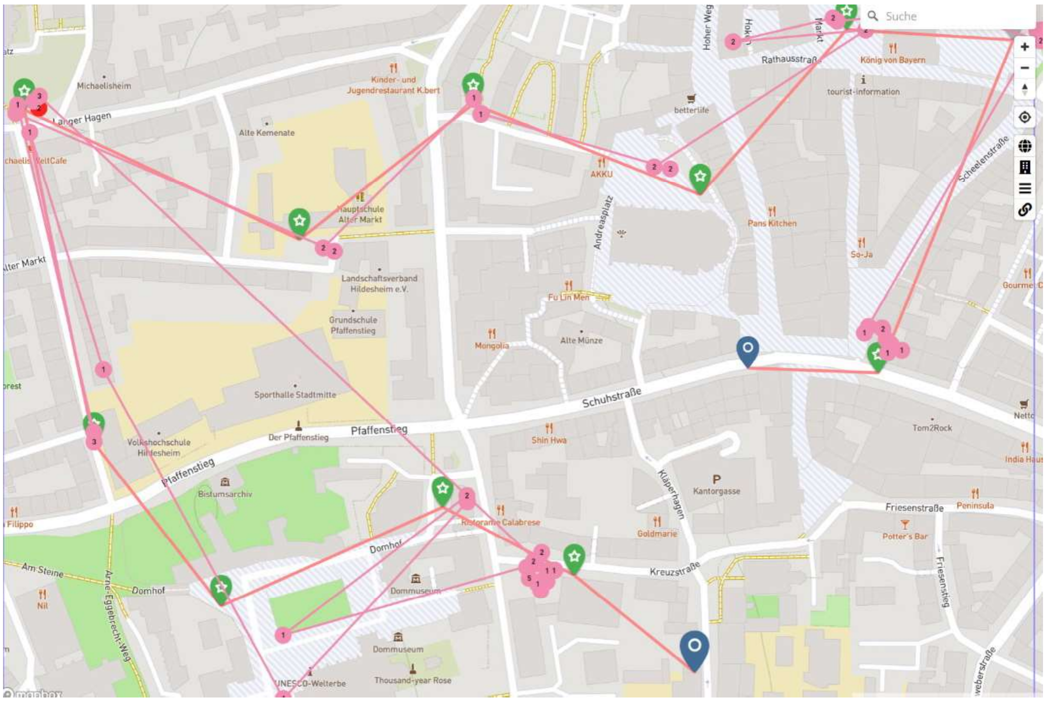

Figure 3: Tracking function in the digital classroom.

OUTLOOK: THE WORKSHOP

Within the proposed workshop, the participants get to know the system from a student's and a teacher's perspective. After a short introduction on outdoor learning and math trails, the participants experience a math trail supported by the MathCityMap app. Subsequently, the participants share their experience with each other.

The second part of the seminar focuses on the teacher's perspective and starts with an introduction of the web portal and the presentation of criteria for meaningful outdoor math tasks. With this basis, the participants create their own tasks in the MathCityMap web portal. The workshop ends with a discussion on the potentials of outdoor mathematics and the benefits of supporting this outdoor experiences by the help of mobile learning.

As preparation for the workshop, the participants should download the MathCityMap app and register in the MathCityMap web portal. They should participate with their computer and have a smartphone/tablet on site. Let's join the community!

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HOW <COLETTE/> FACILITATES TEACHING COMPUTATIONAL THINKING

Rebecca S. Stäter, Tim Läufer and Matthias Ludwig
Goethe University Frankfurt, Germany

Abstract. Although the everyday life of students becomes increasingly digitalized, this process does not necessarily lead to a proficient use and understanding of digital tools. Students must be taught how to use them, but the lack of easy-to-use teaching materials hinders teachers to include Computational Thinking in their lessons. <colette/> is an easy-to-use two-component-system to include Computational Thinking in a variety of school subjects. <colette/> guides teachers to create meaningful tasks and gives students an environment to work on those tasks.

Key words: Computational Thinking, digital teaching, task families.

INTRODUCTION

Computational Thinking (CT) has emerged as a skillset which is needed in everyday life and education (Wing, 2006; Bocconi, 2016). For our purposes CT consists of Abstraction, Algorithmic Thinking, Automation, Debugging, Decomposition, and Generalization (see Bocconi (2016) for a more detailed description of these skills). <colette/> is a free to use web portal for educators to teach CT concepts in an engaging way where students will use the app on their smartphones or tablets. A teacher can create tasks and combine them to learning paths (Roth, 2015), which can then be traversed digitally by the students via an app. Additionally, we want to create opportunities in workshops for educators themselves to learn about CT and how to integrate this in their classroom.

THE <COLETTE/> SYSTEM

The <colette/> system offers a low-threshold approach both for students and educators to learn about and teach CT. The web portal as an authoring tool guides teachers to create meaningful learning paths, the app is user-friendly designed and uses block-based language for coding exercises. As students can work with their own mobile devices like their smartphones, <colette/> makes use of the Bring-Your-Own-Device approach avoiding, hence, the necessity for schools to invest in expensive equipment.

Creation of Paths with Tasks in the Web Portal.

The images and parts of the text of the web portal and the app are also found in the article by Stäter et al. in these proceedings (Stäter et al., 2023).

The web portal is an authoring and path management tool. Tasks can be created by editing Task Families and then combined to a learning path (Roth, 2015). The handbook, a source of information about the web portal, app and CT, is found in the web portal as well.

A task is created by choosing a Task Family as a template (Figure 1.1), choosing an assignment type (Figure 1.2), selecting a scenario (or variation) of the Task Family

Stäter, R. S., Läufer, T., & Ludwig, M. (2023). How <colette/> Facilitates Teaching Computational Thinking. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 197–200). WTM. https://doi.org/10.37626/GA9783959872522.0.25

(Figure 1.3), editing the scenario's settings (Figure 1.4), and finishing the task data such as the title, problem definition (assignment text), task picture, and hints (Figure 1.5).

This process can be repeated to create multiple tasks which can then be combined to a learning path consisting of a title, a description, and the tasks. When creating a path, a code is automatically assigned. This code is used to identify the path and can be entered in the app so the students can work on this path using the app.

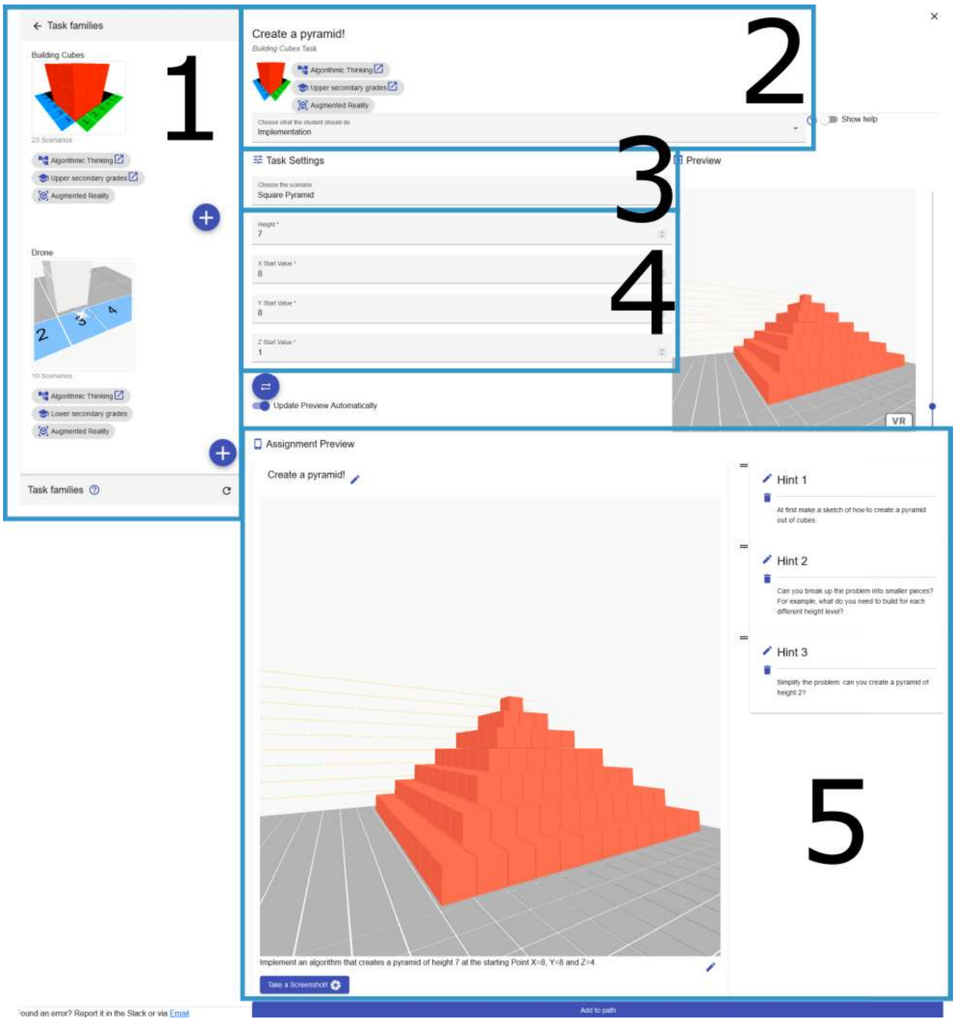

Figure 10: Creating a task in the <colette/> web portal (screenshot). You can see the work flow of an educator going from step 1 to step 5 (labeled by large black numbers) to then save the created task from the "Building Cubes" Task Family. A final path consists of multiple such tasks.

Viewing and Solving a Path in the <colette/> App

After the educator has created a learning path (Roth, 2015), the student can start working on that path by using the app (Figure 2). The included AR view emerges the code into the reality by showing either the coded figure (in the Task Family "Building Cubes") or the coded route (in the Task Family "Drone"). In this way a student can debug their code and see whether the code did what they expected.

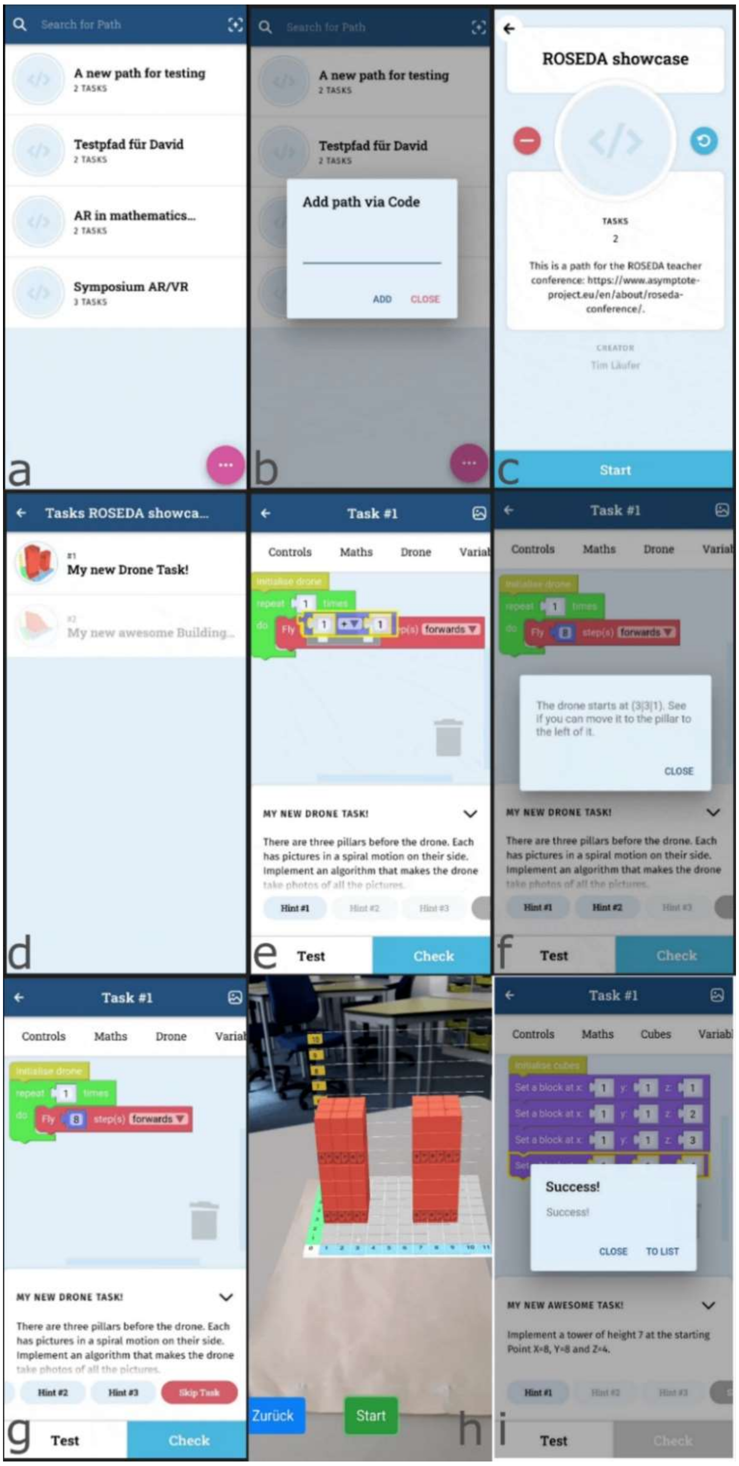

Figure 11: Viewing, solving and reviewing a task with the <colette/> smartphone app (screenshots). Upon starting the app the student can see their already added paths (a), import a new path (b) and start the path while seeing details about the path (c). They will then see the tasks within the path (d) to start working on one (e). Tiered hints will help the student to get to a solution (f) and after having seen all of the hints the student can also skip the task (g). In a AR preview the student can see an animation of the drone flying along the coded route to take pictures (h). When successful a notification is shown (i).

OUTLOOK: THE WORKSHOP

Within the proposed workshop, the participants will get to know the <colette/> system first from the students' point of view by exploring and testing the app, then from the educators' perspective by using the web portal. They will familiarize themselves with the concept "CT" and the creation of CT fostering tasks.

After a short introduction to CT, the participants experience a path using the app. After a short feedback round they will start focusing on how to create a path from the technical perspective as well as the didactical perspective. Both will be covered in the workshop.

As preparation for the workshop, the participants should download the <colette/> app and should bring their own devices (laptop, smartphone) to the workshop, so that we can start creating a <colette/> path.

Acknowledgment

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Supporting education through digital technology becomes increasingly important. Especially in the light of the Covid-19 pandemic, online teaching and learning has been increased massively. In these precarious times, multiple approaches have been developed to enable the delivery of online education. But also for the regular classroom setting, more and more technologies are developed and implemented in educational practice.
This volume contains the papers presented at the *Research On STEM Education in the Digital Age (ROSEDA) Conference*, held in Porto, Portugal, in February 2023. The proceedings summarize and link theoretical considerations, practical experiences and ideas, and empirical research on the use of technology to enrich students' learning. Hereby, the papers focus on the STEM subjects of Mathematics, Technology, Engineering and Mathematics.
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